A TREATISE
OF THE
Natural Grounds, and Principles
OF
HARMONY.

By William Holder, D. D. Fellow of the Royal Society, and late Sub-Dean of their MAJESTY's Chapel Royal.

To which is Added, by way of APPENDIX:
RULES for Playing a Thorough-Bass; with Variety of Proper Lessons, Fuges, and Examples to Explain the said RULES. Also Directions for Tuning an Harpsichord or Spinnet.

By the late Mr. Godfrey Keller.

With several new Examples, which before were wanting, the better to explain some Passages in the former Impressions.

The whole being Revis'd, and Corrected from many gross Mistakes committed in the first Publication of these Rules.

L O N D O N:
Printed by W. Pearson, over against Wright's Coffee-House in Aldersegate-street; for J. Wilcox in Little-Britain; and T. Osborne in Gray's-Inn. 1731.
THE PUBLISHER TO THE READER.

The intention of publishing Mr. Keller's Rules, (by way of Appendix to Dr. Holder's Book) was chiefly to rescue them from many mistakes and errors, which were occasioned by the ignorance of the first Publishers of them on plates, which would never have happened, if the judicious Author had lived to have corrected the plates himself: Nor could he have suffered those examples, which are now added, to have been wanting, the better to explain some of the said rules, which before were only printed in figures, without a proper illustration of the same in musical notes; as is evident by many instances of the same kind throughout the said work.

And as this book may fall into the hands of some, who have (not only a taste for Dr. Holder's treatise, but also) a genius for composing as well as for playing a through bass; it is not improper to observe, that there are many excellent rules contain'd in it, which will be found of great advantage to young composers, as well as to those who practice a through bass; especially with regard to the various ways of taking discords, which is one of the most difficult parts of composition.

And I humbly presume, that for these reasons, it will become at least as useful and instructive, as any thing that has hitherto been published of this kind.

Vale.

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THE
Natural Grounds and Principles
OF
HARMONY.

The INTRODUCTION.

HARMONY consists of Causes, Natural and Artificial, as of Matter and Form. The Material Part of it, is Sound or Voice. The Formal Part is, The Disposition of Sound or Voice into Harmony; which requires, as a preparative Cause, skilful Composition; and, as an immediate Efficient, Artful Performance.

The former Part, viz. The Matter, lies deep in Nature, and requires much Research into Natural Philosophy to unfold it; to find how Sounds are made, and how they are first fitted by Nature for Harmony, before they be disposed by Art: Both together make Harmony compleat.

B

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The INTRODUCTION.

Harmony, then, results from practick Musick, and is made by the Natural and Artificial Agreement of different Sounds, (viz. Grave and Acute) by which the Sense of Hearing is delighted.

This is proper in Symphony, i.e. Consent of more Voices in different Tones; but is found also in solitary Musick of one Voice, by the Observation and Expectation of the Ear, comparing the Habitudes of the following Notes to those which did precede.

Now the Theory in Natural Philosophy, of the Grounds and Reasons of this Agreement of Sounds, and consequent Delight and Pleasure of the Ear, (leaving the Management of these Sounds to the Masters of Harmonick Composure, and the skilful Artists in Performance) is the Subject of this Discourse. The Design whereof (for the Sake and Service of all Lovers of Musick, and particularly the Gentlemen of Their Majesty's Chapel Royal) is, to lay down these Principles as short, and intelligible, as the Subject Matter will bear.

Where the first thing Necessary, is a Consideration of somewhat of the Nature of Sound in General; and then, more particularly, of Harmonick Sounds, &c.

CHAP.
C H A P. I.

Of Sound in General.

In General to pass by what is not pertinent to this Design) Sence and Experience confirm these following Properties of Sound.

1. All Sound is made by Motion, viz. by Percussion with Collision of the Air.

2. That Sound may be propagated, and carried to Distance, it requires a Medium by which to pass.

3. This Medium (to our Purpose) is Air.

4. As far as Sound is propagated along the Medium; so far also the Motion pasheth. For (if we may not say that the Motion and Sound are one and the same thing, yet
yet at least) it is necessarily consequent, that if the Motion cease, the Sound must also cease.

5. Sound, where it meets with no Obstacle, passeth in a Sphere of the Medium, greater or less, according to the Force and Greatness of the Sound; of which Sphere the sonorous Body is as the Centre.

6. Sound, so far as it reacheth, passeth the Medium, not in an Instant, but in a certain uniform Degree of Velocity, calculated by Gassendus, to be about the rate of 276 Paces, in the space of a second Minute of an Hour. And where it meets with any Obstacle, it is subject to the Laws of Reflexion, which is the Cause of Echo's Meliorations, and Augmentations of Sound.

7. Sound, i.e. the Motion of Sound, or founding Motion, is carried through the Medium or Sphere of Activity, with an Impetus or Force, which shakes the free Medium, and strikes and shakes every Obstacle it meets with, more or less, according to the vehemency of the Sound, and Nature of the Obstacle, and Nearness of it to the Centre, or sonorous Body. Thus the
Of Sound in General.

the impetuous Motions of the Sound of Thunder, or of a Canon, shake all before it, even to the breaking of Glass Windows, &c.

8. The Parts of the sounding Body are moved with a Motion of Trembling, or Vibration, as is evident in a Bell or Pipe, and most manifest in the string of a Musical Instrument.

9. This Trembling, or Vibration, is either equal and uniform, or else unequal and irregular; and again, swifter or slower, according to the Constitution of the sonorous Body, and Quality and Manner of of Percussion; and from hence arise Differences of Sounds.

10. The Trembling, or Vibration of the sonorous Body, by which the particular Sound is constituted and discriminated, is impressed upon, and carried along the Medium in the same Figure and Measure, otherwise it would not be the same Sound, when it arrives at a more distant Ear, i.e., the Tremblings and Vibrations (which may be called Undulations) of the Air or Medium, are all along of the same Velocity and Figure, with those of the sonorous Body, by which they are caused.

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The Differences of Sounds, as of one Voice from another, &c. (besides the Difference of Tune, which is caused by the Difference of Vibrations) arise from the Constitution and Figure, and other Accidents of the sonorous Body.

II. If the sonorous Body be requisitely constituted, i.e. of Parts solid, or tense, and regular, fit, being struck, to receive and express the tremulous Motions of Sound, equally and swiftly, then it will render a certain and even Harmonical Tone or Tune, received with Pleasure, and judged and measured by the Ear: Otherwise it will produce an obtuse or uneven Sound, not giving any certain or discernable Tune.

Now this Tune, or Tuneable Sound, φθόνος, i.e. φωνῆς πλοῦς ἵμμελίς ἐπὶ μίαν τόλμην, An agreeable Cadence of Voice, at one Pitch or Tension. This Tuneable Sound (I lay) as it is capable of other Tensions towards Acuteness, or Gravity, i.e. the Tensions greater or less, the Tune graver or more acute, i.e. lower or higher, is the first Matter or Element of Musick. And this Harmonick Sound comes next to be considered.

CHAP.
Of Sound Harmonick.

CHAP. II.

Of Sound Harmonick.

THE first and great Principle upon which the Nature of Harmonical Sounds is to be found out and discovered, is this: That the Tune of a Note (to speak in our vulgar Phrase) is constituted by the Measure and Proportion of Vibrations of the sonorous Body; I mean, of the Velocity of those Vibrations in their Recourses,

For, the frequenter the Vibrations are, the more Acute is the Tune; the fewer and fewer they are in the same Space of Time, by so much the more Grave is the Tune. So that any given Note of a Tune, is made by one certain Measure of Velocity of Vibrations; viz. Such a certain Number of Courses and Recourses. e.g. of a Chord or String, in such a certain Space of Time, doth constitute such a certain determinate Tune. And all such Sounds as are Unisons, or of the same Tune with that given Note, though upon whatsoever different Bodies, (as String, Bell, Pipe, Larynx, &c.) are made with Vibrations or Trem-
Tremblings of those Bodies, all equal each to the other: And whatsoever Tuneable Sound is more Acute, is made with Vibrations more swift; and whatsoever is more grave, is made with more slow Vibrations: And this is universally agreed upon, as most evident to Experience, and will be more manifest through the whole Theory.

And, That the Continuance of the Sound in the same Tune, to the last, (as may be perceived in Wire-strings, which being once struck will hold their Sound long) depends upon the Equality of Time of the Vibrations, from the greatest Range till they come to cease: And this perfectly makes out the following Theory of Consonancy, and Dissonancy.

Some of the ancient Greek Authors of Musick, took Notice of Vibrations: And that the swifter Vibrations caused Acuter, and the slower, graver Tones. And that the Mixture, or not Mixture of Motions creating several Intervals of Tune, was the Reason of their being Concord or Discord. And likewise, they found out the several Lengths of a Monochord, proportioned to the several Intervals of Harmonick Sounds: But they did not make out the Equality of Measure of Time, of the
Of Sound Harmonick.

Vibrations last spoken of, neither could be prepared to answer such Objections, as might be made against the Continuity of the sameness of Tune, during the Continuance of the Sound of a String, or a Bell after it is struck. Neither did any of them offer any Reasons for the Proportions assigned, only it is said, that Pythagorus found them out by Chance.

But now, These (since the Acute Galileo hath observed, and discovered the Nature of Pendulums) are easy to be explained, which I shall do, premising some Consideration of the Properties of the Motions of a Pendulum.

Hang a Plumbet C on a String or Wire, fixed at O. Bear C to A: Then let it range freely, and it will move toward B, and from thence swing back towards A. The Motion from A to B, I call the Course,
Of Sound Harmonick.

Course, and back from B to A, the Recourse of the Pendulum, making almost a Semi-Circle, of which O is the Centre. Then suffering the Pendulum to move of itself, forwards and backwards, the Range of it will at very Course and Recourse abate, and diminish by degrees, till it come to rest perpendicular at O C.

Now that which Galileo first observed, was, that all the Courses and Recourses of the Pendulum, from the greatest Range through all Degrees till it come to rest, were made in equal Spaces of Time. That is, e. g. The Range between A and B, is made in the same Space of Time, with the Range between D and E; the Plumbet moving swifter between A and B, the greater Space; and slower between D and E, the lesser; in such Proportions, that the Motions between the Terms A B and D E, are performed in equal Space of Time.

And here it is to be Noted, that wherever in this Treatise, the swiftness or slowness of Vibrations is spoke of, it must be always understood of the frequency of their Courses and Recourses, and not of the Motion by which it paffeth from one side to another. For it is true, that the same Pendulum under the same Velocity of Returns,
Of Sound Harmonick:

moves from one side to the other, with greater or less Velocity, according as the Range is, greater or less.

And hence it is, that the Librations of a Pendulum are become so excellent, and useful a Measure of Time; especially when a second Observation is added, that, as you shorten the Pendulum, by bringing C nearer to its Centre O, so the Librations will be made proportionably in a shorter Measure of Time, and the contrary if you lengthen it. And this is found to hold in a Duplicate proportion of length to Velocity. That is, the length quadrupled, will subduple the Velocity of Vibrations: And the Length subquadrupled, will duple the Vibrations, for the Proportion holds reciprocally. As you add to the length of the Pendulum, so you diminish the frequency of Vibrations, and increase them by shortening it.

Now therefore to make the Courses of a Pendulum doubly swift, i.e. to move twice in the same Space of Time, in which it did before move once; you must subquadruple the Length of it, i.e. make the Pendulum but a quarter so long as it was before. And to make the Librations doubly slow, to pass once in the Time they did
did pass twice; you must quadruple the Length; make the Pendulum four Times as long as it was before, and so on in what Proportion you please.

Now to apply this to Musick, make two Pendulums, $AB$ and $CD$, fasten together the Plumbets $B$ and $D$, and stretch them at length, (fixing the Centers $A$ and $C$.) Then, being struck, and put into Motion; the Vibrations, which before were distinct, made by $AB$, and $CD$, will now be united (as of one entire String) both backward and forward, between $E$ and $F$. Which Vibrations (retaining the aforesaid Analogy to a Pendulum) will be made in equal Spaces of Time, from the first to the last; i.e. from the greatest Range to the least, until they cease. Now, this being a double Pendulum, to subduple the swiftness of the Vibrations, you do but double the length from $A$ to $C$, which will be quadruple to $AB$. The lower Figure is the same with that above, only the Plummets taken off.
And here you have the Nature of the String of a Musical Instrument, resembling a double *Pendulum* moving upon two Centers, the Nut and the Bridge, and Vibrating with the greatest Range in the middle of its Length; and the Vibrations equal even to the last, which must make it keep the same Tune so long as it Sounds. And because it doth manifestly keep the same Tune to the last; it follows that the Vibrations are equal, confirming one another by two of our Senses; in that we see the Vibrations of a *Pendulum* move equally, and we hear the Tune of a String, when it is struck, continue the same.

The Measure of swiftness of Vibrations of the String or Chord, (as hath been said,) constitutes and determines the Tune, as to the Acuteness and Gravity of the Note which it sounds; And the lengthning or short-
shortening of the String, under the same Tension, determines the Measure of the Vibrations which it makes. And thus, Harmony comes under mathematical Calculations of Proportions, of the length of Chords, of the Measure of Time in Vibrations; of the Intervals of Tuned Sounds. As the length of one Chord to another, *Cateris paribus*, I mean, being of the same Matter, thickness and tension; so is the Measure of the Time of their Vibrations. As the Time of Vibrations of one String to another, so is the Interval or Space of Acuteness or Gravity of the Tune of that one, to the Tune of the other: And consequently, as the length is (*Cateris paribus*) so is the determinate Tune.

And upon these Proportions in the Differences of Lengths of Vibrations, and of Acuteness and Gravity; I shall insist all along in this Treatise, very largely and particularly, for the full Information of all such ingenious Lovers of Musick, as shall have the Curiosity to inquire into the Natural Causes of Harmony, and of the *Phenomena* which occur therein, though otherwise, to the more learned in Musick and Mathematical Proportions, all might be expressed very much shorter, and still be more shortened by the help of Symbols.

And
AND here we may fix our Foot: Concluding, that what is evident to Sense, of the Phenomena, in a Chord, is equally (though not so discernably) true of the Motions of all other Bodies which render a tuneable Sound, as the Trembling of a Bell or Trumpet, the forming of the Larynx in our selves, and other Animals, the throat of Pipes and of those of an Organ, &c. All of them in several Proportions sensibly trembling and impressing the like Undulations of the Medium, as is done by the several (more manifest) Vibrations of Strings or Chords.

In these other Bodies, last spoken of, we manifestly see the Reason of the Difference of the swiftness of their Vibrations (though we cannot so well measure them) from their Shape, and other Accidents in their Constitution; and chiefly from the Proportions of their Magnitudes; the Greater generally Vibrating slower, and the Less more swiftly, which give the Tunes accordingly. We see it in the Greatness of a String; a greater and thicker Chord will give a graver and lower Tone, than one that is more slender, of the same Tension and Length; but they may be made Unison by altering their Length and Tension.
Tension is proper to Chords or Strings (except you will account a Drum for a Musical Instrument, which hath a Tension not in Length, but in the whole surface) as when we wind up, or let down the Strings, i.e. give them a greater or less Tension, in tuning a Viol, Lute, or Harpsichord, and is of great Concern, and may be measured by hanging Weights on the String to give it Tension but not easily, nor so certainly.

But the lengths of Chords (because of their Analogy to a Pendulum) is chiefly considered, in the discovery of the Proportions which belong to Harmony, it being most easie to measure and design the Parts of a Monochord, in relation to the whole String; and therefore all Intervals in Harmony may first be described, and understood, by the Proportions of the length of Strings, and consequently of their Vibrations. And it is for that Reason, that in this Treatise of the Grounds of Harmony, Chords come so much to be considered, rather than other sounding Bodies, and those, chiefly in their Proportions of Length. It is true, that in Wind-Instruments, there is a Regard to the Length of Pipes, but they are not so well accommodated (as our Chords) to be examined, nei-
neither are their Vibrations, nor the measure of them so manifest.

There are some Musical Sounds which seem to be made, not by Vibrations but by Pulses as by whisking swiftly over some Silk or Camblet-fluffs, or over the Teeth of a Comb, which render a kind of Tune more Acute or Grave, according to the swiftness of the Motion. Here the Sound is made, not by Vibrations of the same Body, but by Percussion of several equal, and equidistant Bodies; as Threads of the Stuff, Teeth of the Comb passing over them with the same Velocity as Vibrations are made. It gives the same Modification to the Tune, and to the Undulations of the Air, as is done by Vibrations of the same Measure; the Multiplicity of Pulses or Percussions, answering the Multiplicity of Vibrations. I take this Notice of it, because others have done so; but I think it to be of no use in Musick.
APPENDIX.

Before I conclude this Chapter, it may seem needful, better to confirm the Foundation we have laid, and give the Reader some more ample Satisfaction about the Motions and Measures of a Pendulum, and the Application of it to Harmonick Motion.

FIRST then, it is manifest to Sense and Experience, and out of all dispute; that the Courses and Recourses return sooner or later, i.e. more or less frequently, according as the Pendulum is shortened, or made longer. And that the Proportion by which the Frequency increaseth, is (at least) very near duplicate, viz. of the length of the Pendulum, to the Number of Vibrations, but is in reverse, i.e. as the Length encreaseth, so the Vibrations decrease; and contrary, quadruple the Length, and the Vibrations will be subduplicated. Subquadruple the Length, and the Vibrations will be dupled. And lastly, that the Librations, the Courses and Recourses of the same Pendulum, are all made in equal Space of Time, or very near to it, from the greatest Range to the least.
least. Now though the duplicate Proportion assigned, and the equality of Time, are a little called in question, as not perfectly exact, though very near it; yet in a Monochord we find them perfectly agree, viz. as to the length, Duple instead of Duplicate, because a String fastned at both ends is as a double Pendulum, each of which is quadrupled by dupling the whole String. And on this duple Proportion, depend all the Rations found in Harmony. And again, the Vibrations of a String are exactly equal, because they continue to give the same Tune.

Supposing then some little difference may sometime seem to be found in either of these Motions of a Pendulum, yet the nearness to Truth is enough to support our Foundation, by shewing what is intended by Nature, though it sometimes meet with secret Obstacles in the Pendulum, which it does not in a well made String. We may justly make some Allowance for the Accidents, and unseen Causes, which happen to make some little Variations in Trials of Motion upon gross Matter, and consequently the like for nicer Experiments made upon the Pendulum. It is difficult to find exactly the determinate Point of the Plumbet, which regulates the Motions of
of the Pendulum, and consigns its just length. Then observe the Varieties which happen through various sorts of Matter, upon which Experiments are made. Mersennius tells us, that heavier Weights of the same length move slower, so that whilst a Lead Plumbet makes 39 Vibrations, Cork or Wood will make at least 40.

Again, that a stiff Pendulum vibrates more frequently, than that which hangs upon a Chord. So that a Bar of Iron, or Staff of Wood ought to be half as long again as the other, to make the Vibrations equal. Yet in each of these respectively to itself, you will find the duplicate Proportion and Equality of Vibration, or as near as may be. And (as to Equality) though in the Extremes of the Ranges of Librations, viz. the greatest compared to the least, there may (from unseen Causes) appear some Difference, yet there is no discernable Difference of the Time of Vibrations of a Pendulum in Ranges, that are near to one another, whether greater or less; which is the Case of the Ranges of the Vibration of a String being made in a very small Compass: And therefore the Librations of a Pendulum, limited to a small Difference of Ranges, do well correspond with the Vibrations of a String.
As to Strings, the Whole of Harmony depends upon this experimented and unquestioned Truth, that Diapason is duple to its Unison, and consequently Diapente is Sesquialterum, Diatesseron Sesquiquartum, &c. Yet if you happen to divide a faulty String of an Instrument, you will not find the Octave just in the Middle, nor the other Intervals in their due Proportion, which is no default in Nature, but the Matter we apply. A false String is that, which is thicker in one Part of its Length, than in another. The thicker Part naturally vibrates slower, and sounds graver; the more slender Part vibrates swifter, and sounds more acute. Thus whilst two Sounds so near one another, are at once made upon the same String, they make a rough discording Jarr, being a hoarse Tune mixed of both, more or less, as the String is more or less unequal: And if the thicker Part be next the Frets, then the Fret (for Example D. F. H. &c. in a Viol or Lute) will render the Tune of the Note too sharp; and the contrary, if the slender Part of the String be next the Frets; because in the former, the thicker Part is stopped, and the thinner sounds more of the acuter Part of this unhappy Mixture: As in the later, the thicker Part is left to found the graver Tune, and thus the Fret will
will give a wrong Tune though the Fault be not in the Fret, but in the String; which yet, by an unwary Experimenter, may happen to cause the *Sezio Canonis* to be called in question, as well as the Measures of a *Pendulum* are disputed.

**But** all this does not disprove the Measures found out, and assigned to Harmonick Intervals, which are verified upon a true String or Wire as to their Lengths, and as to the Equality of Recourse in their Vibrations, though *Pendulums* are thought to move slower in their least Ranges; yet, as to Strings, in the very small Ranges which they make, (which are much less in other Instruments, or sounding Bodies) I need add no more than this, that the Continuance of the same Tune to the last, after a Chord is struck, and the continued Motion in less Vibrations of a sympathizing String, during the Continuance of greater Vibrations of the String which is struck, do either of them sufficiently demonstrate, that those greater or less Vibrations, are both made in the same Measure of Time, according to their Proportions, keeping exact Pace with each other. Otherwise; In the former, the Tune would sensibly alter, and in the latter, the sympathizing String could not be continued in
in its Motion. This was not so well concluded, till the late Discoveries of the Pendulum gave light to it.

There is one thing more which I must not omit. That the Motions of a Pendulum, may seem not so proper to explicate the Motions of a String, because the said Motions depend upon differing Principles, viz. those of a Pendulum upon Gravity; those of a String upon Elasticity. I shall therefore endeavour to shew, how the Motions of a Pendulum, agree with those of a Spring, and how properly the Explication of the Vibrations of a String, is deduced from the Properties of a Pendulum.

The Elastick power of a Spring, in a Body indued with Elasticity, seems to be nothing else, but a natural Propension and Endeavour of that Body, forced out of its own Place, or Posture, to restore itself again into its former, more easie and natural Posture of Rest. And this is found in several Sorts of Bodies, and makes different Cases, of which I shall mention some.

If the Violence be by Compression, forcing a Body into less room than it naturally requires; then the Endeavour of Resti-
Situation, is by Dilatation to gain room enough. Thus Air may be compressed into less Space, and then will have a great Elasticity, and struggle to gain its room. Thus, if you squeeze a dry Sponge, it will naturally, when you take off the Force, spread itself, and fill its former Place. So, if you press with your Finger a blown Bladder, it will spring and rise again to its Place. And to this may be reduced the Springs of a Watch, and of a Spiral Wire, &c.

Again, a stiff, but pliable Body, fastened at one End, and drawn aside at the other, will spring back to its former Place; this is the Case of Steel-springs of Locks, Snap-haunces, &c. and Branches of Trees, when shaken with the Wind, or pulled aside, return to their former Posture: As is said of the Palm, Depressa Resurgo. And there are innumerable instances of this kind, where the force is by bending, and the Restitution by unbending or returning.

This kind is resembled by a Pendulum, or Plummets hanging on a String, whose Gravity, like the Spring in those other Bodies, naturally carries it to its place, which here is downward; all heavy Bodies naturally descending till they meet with some Ob-
Obstacle to rest upon. And the lowest that the Plummet can descend in its Restraint by the String, is, when it hangs perpendicular, as to $AB$; where it is nearest to the Horizontal Plane $GH$, and therefore lowest. Now, if you force the Plummet upward (held at length upon the String) from $B$ to $C$, and let it go; it will, by a Spontaneous Motion, endeavour its Restitution to $B$: But, having nothing to stop it but Air, the Impulse of its own Velocity will carry it beyond $B$, towards $D$; and so backward and forward, decreasing at every Range, till it come to rest at $B$.

Thus the Pendulum and Spring agree in Nature, if you consider the Force against them, and their Endeavour of Restitution.
But further, if you take a thin stiff Laminæ of Steel, like a Piece of Two-penny Riband of some length, and nail it fast at one End, (the remainder of it being free in the Air) then force the other End aside and let it go; it will make Vibrations backward and forward, perfectly answering those of a Pendulum. And much more, if you contrive it with a little Steel Button at the End of it, both to help the Motion when once set on foot, and to bear it better against the Resistance of the Air. There will be no difference between the Vibrations of this Spring, and of a Pendulum, which in both, will be alike increased or decreased in Proportion to their Lengths. The same End (viz. Rest) being, in the same manner, obtained by Gravity in one, and Elasticity in the other.

Further yet, if you nail the Spring above, and let it hang down perpendicular, with a heavier Weight at the lower End, and then set it on moving, the Vibrations will be continued and carried on both by Gravity and Elasticity, the Pendulum and the Spring will be most friendly joyned to cause a simple equal Motion of Librations, I mean, an equal Measure of Time in the Recourses; only the Spring answerable to its Strength, may cause the
Librations to be some what swifter, as an Addition of Tension does to a String continued in the same length.

I come now to consider a String of an Instrument, which is a Spring fastned at both Ends. It acquireth a double Elasticity. The first by Tension, and the Spring is stronger or weaker, according as the Tension is greater or less. And by how much stronger the Spring is, so much more frequent are the Vibrations, and by this Tension therefore, the Strings of an Instrument keeping the same length are put in Tune, and this Spring draws length-ways, endeavouring a Relaxation of the Tension.

But then, Secondly, the String being under a stated Tension, hath another Elastic Power side-ways, depending upon the former, by which it endeavours, if it be drawn aside, to restore it self to the easlest Tension, in the shortest, viz. straightest line.

In the former Case, Tension doth the same with abatement of length, and affects the String properly as a Spring, in that the String being forcibly stretched, as forcibly draws back to regain the remiss Posture in which it was before: And bears little
little Analogy with the Pendulum, except in general, in their spontaneous Motions in order to their Restitution.

But there is great Correspondence in the second Case, between the Librations of a Pendulum and the Vibrations of a String (for so, for distinctions sake, I will now call them) in that they are both proportioned to their length, as has been shewn; and between the Elasticity which moves the String, and Gravity which moves the Pendulum, both of them having the same Tendency to Restitution, and by the same Method. As the Declivity of the Motion of a Pendulum, and consequently the Impulse of its Gravity is still lessened in the Arch of its Range from a Semi-Circle, till it come to rest perpendicular; the Descent thereof being more downright at the first and greatest Ranges, and more Horizontal at the last and shortest Ranges, as may be seen in the preceding Figure $C I E E B$; so the Impulse of Spring is still gradually lessened as the Ranges shorten, and as it gains of relaxation, till it come to be restored to rest in its shortest Line. And this may be the Cause of the Equality of Time of the Librations of a Pendulum, and also of the Vibrations of a String. Now, the Proportions of Length, to the Veloc-
APPENDIX.

Velocity of Vibrations in one, and of Librations in the other, we are sure of, and find by manifest Experience to be quadruple in one, and duple in the other.

Now tack two equal Pendulums together (as before) being fastned at both Ends, take away the Plumbets, and you make it a String, retaining till the same Properties of Motion, only what was said before to be caused by Gravity, must now be said to be done by Elasticity. You see what an easie Step here is out of one into the other, and what Agreement there is between them. The Phanomena are the same, but difficultly experimented in a String, where the Vibrations are too swift to fall under each exact Measure; but most easie in a Pendulum, whose slow Librations may be measured at pleasure, and numbered by distant Moments of Time.

To bring it nearer, make your Tension of the String by Gravity, instead of screwing it up with a Pegg or Pin: Hang weight upon a Pulley at one End of the String, and as you increase the Weight, so you do increase the Tension, and as you increase the Tension, so you increase the Velocity of Vibrations. So the Vibrations are proportionably regulated immediately by Tension.
APPENDIX.

fion, and mediately by Gravity. So that Gravity may claim a share in the Measures of these Harmonick Motions.

But to come still nearer, and home to our purpose. Fasten a Gut or Wire-string above, and hang a heavy Weight on it below, as the Weight is more or less, so will be the Tension, and consequently the Vibrations. But let the same Weight continue, and the String will have a stated setled Tension. Here you have both in one, a Pendulum, and the Spring of a String, which resembles a double Pendulum: Draw the Weight aside, and let it range, and it is properly a Pendulum, librating after the Nature of a Pendulum. Again, when the Weight is at rest, strike the String with a gentle Plectrum made of a Quill, on the upper part, so as not to make the Weight move, and the String will vibrate, and give its Tune, like other Strings fastened at both Ends, as this is also, in this Case. So you have here both a Pendulum and a String, or either, which you please. And (the Tension being supposed to be settled under the same Weight) the common Measure and Regulator of the Proportions of them is the Length, and as you alter the Length, so proportionably you alter at once the Velocity in the Recourfes of the Vi-
APPENDIX.

The Vibrations of the String, and of the Librations of the Pendulum. And though the Vibrations be so much swifter, and more frequent than the Librations, yet the Rations are altered alike. If you subduple the Length of the String, then the Vibrations will be dupled. And if you subquadruple it, then the Librations will be also dupled, allowing for so much of the Body of the Weight as must be taken in, to determine the Length of the Pendulum.

The Vibrations are altered in duple Proportion to the Librations, because (as has been shewn) the String is as a double Pendulum, either one of which supposed Pendulums is but half so long as the String, and is quadrupled by dupling the whole String. Stili therefore the Proportion of their Alterations holds so certainly and regularly with the Proportion of every Change of their common Length, that, if you have the Comparative Ration of either of these two, \textit{viz.} Vibrations or Librations to the Length, you have them both: The increase of the Velocity of Librations being subduple to the increase of the Velocity of Vibrations. And thus the Motions of a Pendulum do fully and properly discover to us, the Motions of a String, by the manifest Correspondence of their
their Properties and Nature. The Pendulum's Motion of Gravity, and the Strings of Elasticity bearing to certain Proportions according to Length, that the Principles of Harmony, may be very properly made out, and most easily comprehended, as explained by the Pendulum. And we find, that in all Ages, this part of Harmony was never so cheerfully understood, as since the late Discoveries about the Pendulum.

And I chuse to make this Illustration by the Pendulum, because it is so easy for Experiment, and for our Comprehension; and the Elastick Power so difficult.

Having seen the Origine of Tuneable or Harmonick Sounds, and of their Difference in respect of Acuteness and Gravity: It is next to be considered, how they come to be affected with Consonancy and Dissonancy, and what these are.
CHAP. III.

Of Consonancy and Dissonancy.

Consonancy and Dissonancy are the Result of the Agreement, mixture or uniting (or the contrary) of the undulated Motions of the Air or Medium, caused by the Vibrations by which the Sounds of distinct Tunes are made. And those are more or less capable of such Mixture or Coincidence according to the Proportion of the Measures of Velocity in which they are made, i.e. according as they are more or less commensurate. This I might well set down as a Postulatum. But I shall by several Instances endeavour to illustrate the undulating Motions or Undulations of the Air; and confirm what is said of their Agreements and Disagreements. And first the Undulations, by somewhat we see in other Liquids.

Let a Stone drop into the Middle of a small Pond of standing Water when it is quiet, you shall see a Motion forthwith impressed upon the Water, passing and dilating from that Center where the Stone fell, in circular Waves one within an other.
still propagated from the Center, spreading till they reach and dash against the Banks, and then returning, if the force of the Motion be sufficient, and meeting those inner Circles which pursue the same Course, without giving them any Check.

And if you drop a Stone in another place, from that Centre will likewise spread round Waves; which meeting the other, will quickly pass them, each moving forwards in its own proper Figure.

The like is better experimented in Quick-silver, which being a more dense Body, continues its Motions longer, and may be placed nearer your Eye. If you try it in a pretty large round Vessel, suppose of a Foot Diameter, the Waves will keep their own Motion forward and backward, and quietly pass by one another as they meet. Something of this may be seen in a long narrow Passage, where there is not room to advance in Circles.

Make a wooden Trough or long Box, suppose of two Inches broad, and two deep, and twenty long. Fill in three Quarters or half full of quick-silver, and place it Horizontally, when it is at quiet, give it with your Finger a little pat at one End, and
and it will impress a Motion of a ridged Wave a cross, which will pass on to the other End, and dashing against it, return in the same Manner, and dash against the hether End, and go back again, and thus backward and forward, till the Motion cease. Now if after you have set this Motion on foot, you cause such another, you shall see each Wave keep its regular Course; and when they meet one another, pass on without any Reluctancy.

I do not say these Experiments are full to my purpose, because these being upon single Bodies, are not sufficient to express the Disagreements of Disproportionate Motions caused by different Vibrations of different sounding Bodies; but these may serve to illustrate those invisible Undulations of Air. And how a Voice reflected by the Walls of a Room, or by Eccho being of adequate Vibrations, returns from the Wall, and meets the commensurate Undulations passing forwards, without hinder ing one another.

But there are Instances which further confirm the Reasons of Consonancy and Diffonancy, by the manifest agreeing or disagreeing Measures of Motions already spoken of.
It hath been a common Practice to imitate a Tabour and Pipe upon an Organ. Sound together two discording Keys (the base Keys will shew it best, because their Vibrations are slower) let them, for Example, be Gamut with Gamut sharp, or F Faut sharp, or all three together. Though these of themselves should be exceeding smooth and well voyced Pipes; yet, when struck together, there will be such a Battel in the Air between their disproportioned Motions, such a Clatter and Thumping, that it will be like the beating of a Drum, while a Jigg is played to it with the other hand. If you cease this, and sound a full Close of Con cords, it will appear surprizingly smooth and sweet, which shews how Discords well placed, set off Conords in Composition. But I bring this Instance to shew, how strong and vehement these undulating Motions are, and how they correspond with the Vibrations by which they are made.

It may be worth the while, to relate an Experiment upon which I happened. Being in an Arched sounding Room near a shrill Bell of a House Clock, when the Alarm struck, I whittled to it, which I did with ease in the same Tune with the Bell, but, endeavouring to whistle a Note higher
higher or lower, the Sound of the Bell and its cross Motions were so predominant, that my Breath and Lips were check’d so, that I could not whistle at all, nor make any Sound of it in that discords Tunes. After, I founded a thrill whistling Pipe, which was out of Tune to the Bell, and their Motions so clashed, that they seemed to sound like switching one another in the Air.

GALILEO, from this Doctrine of Pendulums, easily and naturally explains the so much admired sympathy of Consonant strings; one (though untouch’d) moving when the other is struck. It is perceptible in Strings of the same, or another Instrument, by trembling so as to shake off a Straw laid upon the other String: But in the same Instrument, it may be made very visible, as in a Bass-Viol. Strike one of the lower Strings with the Bow, hard and strong, and if any of the other Strings be Unison or Octave to it, you shall plainly see it vibrate, and continue to do so, as long as you continue the Stroke of your Bow, and, all the while, the other Strings which are dissonant, rest quiet.

The Reason hereof is this. When you strike your String, the Progressive sound of
Of Consonancy

Of it strikes and starts all the other Strings, and every of them makes a Movement in its own proper Vibration. The Consonant string, keeping measure in its Vibrations with those of the sounding String hath its Motion continued, and propagated by continual agreeing Pulses or Stokes of the other. Whereas the Remainder of the Dissonant strings having no help, but being checked by the Cross Motions of the sounding String, are constrained to remain still and quiet. Like as, if you stand before a Pendulum, and blow gently upon it as it passeth from you, and so again in its next Courses keeping exact time with it, it is most easily continued in its Motion. But if you blow irregularly in Measures different from the Measure of the Motion of the Pendulum, and so most frequently blow against it, the Motion of it will be so checked, that it must quickly cease.

And here we may take Notice, (as has been hinted before) that this also confirms the aforesaid Equality of the Time of Vibrations to the last, for that the small and weak Vibrations of the sympathizing String are regulated and continued by the Pulses of the greater and stronger Vibrations of the sounding String, which proves, that
notwithstanding that Disparity of Range, they are commensurate in the Time of their Motion.

This Experiment is ancient: I find it in Aristides Quintilianus a Greek Author, who is supposed to have been contemporaneous with Plutarch. But the Reason of it deduced from the Pendulum, is new, and first discovered by Galileo.

It is an ordinary Trial, to find out the Tune of a Beer-glass without striking it, by holding it near your Mouth, and humming loud to it, in several single Tunes, and when you at last hitt upon the Tune of the Glass, it will tremble and Echo to you. Which shews the Consent and Uniformity of Vibrations of the same Tune, though in several Bodies.

To close this Chapter. I may conclude that Consonancy is the Passage of several Tuneable sounds through the Medium, frequently mixing and uniting in their undulated Motions, caused by the well proportioned commensurate Vibrations of the sonorous Bodies, and consequently arriving smooth, and sweet, and pleasant to the Ear. On the contrary, Dissonancy is from disproportionate Motions of Sounds, not mixing.
Of Consonancy

ing, but jarring and clashing as they pass, and arriving to the Ear Harsh, and Grating, and Offensive. And this, in the next Chapter shall be more amply explained.

Now, what Conords and Discords are thus produced, and in use, in order to Harmony, I shall next consider.

CHAP. IV.

Of Conords.

Conords are Harmonick sounds, which being joined please and delight the Ear; and Discords the Contrary. So that it is indeed the judgment of the Ear that determines which are Conords and which are Discords. And to that we must first resort to find out their Number. And then we may after search and examine how the natural Production of those Sounds, disposeth them to be pleasing or unpleasant. Like as the Palate is absolute Judge of Tasts, what is sweet, and what is bitter, or sour, &c. though there may be also found out some natural Causes of those Qualities. But the Ear being entertained with Motions which fall under exact De-
monstrations of their Measures, the Doctrine hereof is capable of being more accurately discovered.

First then, (setting aside the Unison Concord, which is no Space nor Interval, but an Indentity of Tune) the Ear allows and approves these following Intervals, and only these for Concords to any given Note, viz. the Octave or Eighth, the Fifth, then the Fourth, (though by later Masters of Musick degraded from his Place) then the Third Major, the Third Minor, the Sixth Major, and the Sixth Minor. And also such, as in the Compass of any Voice or Instrument beyond the Octave, may be compounded of these, for such those are, I mean compounded, and only the former seven are simple Concords; not but that they may seem to be compounded, viz. the greater of the less within an Octave, and therefore may be called Systems, but they are Originals. Whereas beyond an Octave, all is but Repetition of these in Compound with the Eighth, as a Tenth is an Eighth and a Third; a Twelfth is an Eighth and a Fifth; a Fifteenth is Diapason, i.e. two Octaves, &c.

But notwithstanding this Distinction of Original and Compound Concords; and tho'
Tho' these compounded concords are found, and discerned by their habitude to the original concords comprehended in the system of diapason; (as a tenth ascending is an octave above the third, or a third above the octave; a twelfth is an octave to the fifth, or a fifth to the eighth, a fifteenth is an eighth above the octave, i.e. dildiapason two eighths, &c.) yet they must be own'd, and are to be esteem'd good and true concords, and equally useful in melody, especially in that of comfort.

The system of an eighth, containing seven intervals, or spaces, or degrees, and eight notes reckoned inclusively, as expressed by eight chords, is called diapason, i.e. a system of all intermediate concords, which were anciently reputed to be only the fifth and the fourth, and it comprehends them both, as being compounded of them both: And now, that the thirds and sixths are admitted for concords, the eighth contains them also: viz. a third major and sixth minor, and again a third minor and sixth major. The octave being but a replication of the unison, or given note below it, and the same, as it were in miniature, it closeth and terminates the first perfect system, and
and the next Octave above it ascends by the same Intervals, and is in like manner compounded of them, and so on, as far as you can proceed upwards or downwards with Voices or Instruments, as may be seen in an Organ, or Harpsichord. It is therefore most justly judged by the Ear, to be the chief of all Concords, and is the only Consonant System, which being added to it itself, still makes Concords.

And to it all other Concords agree, and are Consonant, though they do not all agree to each other; nor any of them make a Concord if added to it itself, and the Complement or Residue of any Concord to Diapason, is also Concord.

The next in Dignity is the Fifth, then the Fourth, Third Major, Third Minor, Sixth Major, and lastly Sixth Minor; all taken by Ascent from the Unison or given Note.

By Unison is meant, sometimes the Habitute or Ration of Equality of two Notes compared together, being of the very same Tune. Sometimes (as here) for the given single Note to which the Distance, or the Rations of other Intervals are compared. As, if we consider the Relations to Gamut, to
to which are a Tone or Second, B mi
a Third, C a Fourth, D a Fifth, &c. We
call Gamut the Unison, for want of a more
proper Word. Thus C fa ut, or any other
Note to which other Intervals are taken,
may be called the Unison.

And the Reader may easily discern, in
which Sense it is taken all along by the
Coherence of the Discourse.

I come now to consider the natural Rea-
sons, why Concords please the Ear, by exa-
mining the Motions by which all Con-
cords are made, which having been gene-
really alleged in the beginning of the third
Chapter, shall now more particularly be
discussed.

And here I hope the Reader will par-
don some Repetition in a Subject that stands
in need of all Light that may be, if, for
his ease and more steady Progress, before I
proceed, I call him back to a Review and
brief Summary of some of those Notions,
which have been premis'd and consider-
ed more at large. I have shewed,

1. That Harmonick Sound or Tune
is made by equal Vibrations or Tremblings
of a Body fitly constituted.

2. That
Of Conords.

2. That those Vibrations make their Courses and Recourses in the same Measure of Time; from the greatest Range to the lesser, till they come to rest.

3. That those Vibrations are under a certain Measure of Frequency of Courses and Recourses in a given Space of Time.

4. That if the Vibrations be more frequent, the Tune will be proportionably more Acute; if less frequent, more Grave.

5. That the Librations of a Pendulum become doubly frequent, if the Pendulum be made four times shorter; and twice flower, if the Pendulum be four times longer.

6. That a Chord, or String of a Musical Instrument, is as a double Pendulum, or two Pendulums tacked together at length, and therefore hath the same Effects by dupling; as a Pendulum by quadrupling, i.e. by dupling the Length of the Chord, the Vibrations will be subdued, i.e. be half so many in a given Time. And by subdupling the length of the Chord, the Vibrations will be dupled, and proportionably so in all other Measures of Length, the Vibrations bearing a Reciprocal proportion to the Length.

7. That
7. That these Vibrations impress a Motion of Undulation or Trembling in the Medium (as far as the Motion extends) of the same Measure with the Vibrations.

8. That if the Motions made by different Chords be so commensurate, that they mix and unite; bear the same Course either altogether, or alternately, or frequently: Then the Sounds of those different Chords, thus mixing, will calmly pass the Medium, and arrive at the Ear as one Sound, or near the same, and so will smoothly and evenly strike the Ear with Pleasure, and this is Consonancy, and from the want of such Mixture is Dissonancy. I may add, that as the more frequent Mixture or Coincidence of Vibrations, render the Conords generally so much the more perfect: So, the less there is of Mixture, the greater and more harsh will be the Discord.

From the Premises, it will be easie to comprehend the natural Reason, why the Ear is delighted with those forenamed Conords; and that is, because they all unite in their Motions often, and at the least at every sixth Course of Vibration, which appears from the Rations by which they are constituted, which are all contained
ed within that Number, and all Rations contained within that Space of Six, make Conords, because the Mixture of their Motions is answerable to the Ration of them, and are made at or before every Sixth Course. This will appear if we examine their Motions. First, how and why the Unisons agree so perfectly; and then finding the Reason of an Octave, and fixing that, all the rest will follow.

To this purpose, strike a Chord of a sounding Instrument, and at the same Time, another Chord supposed to be in all respects Equal, \( i.e. \) in Length, Matter, Thickness and Tension. Here then, both the Strings give their Sound; each Sound is a certain Tune; each Tune is made by a certain Measure of Vibrations; the same Vibrations are impressed upon, and carried every way along the Medium, in Undulations of the same Measure with them, until the Sounds arrive at the Ear. Now the Chords being supposed to be equal in all respects; it follows, that their Vibrations must be also equal, and consequently move in the same Measure, joyning and uniting in every Course and Recourse, and keeping till the same Equality, and Mixture of Motions of the String, and in the Medium. Therefore the Habitude of these two Strings
Strings is called Unison, and is so perfectly consonant, that it is an identity of tune, there being no interval or space between them. And the ear can hardly judge, whether the sound be made by two strings, or by one.

But consonancy is more properly considered, as an interval, or space between tones of different acuteness or gravity. And amongst them, the most perfect is that which comes nearest to Unison, (I do not mean betwixt which there is the least difference of interval; but, in whose motions there is the greatest mixture and agreement next to Unison. The motions of two Unisons are in ration of 1 to 1, or of equality. The next ration in whole numbers is 2 to 1, Duple. Divide a monochord in two equal parts, half the length compared to the whole, being in subduple ration, will make double vibrations, making two recourses in the same time that the other makes one, and so uniting and mixing alternately, i.e. every other motion. Then comparing the sounds of these two, and the half will be found to sound an octave to the whole chord. Now the octave (ascending from the Unison) being thus found and fixed to be in duple proportion of vibrations, and
and subduple of Length; consequently the Proportions of all other Intervals are easily found out.

They are found out by resolving or dividing the Octave into the Mean Rations which are contain’d in it. Euclid, in his Secund Canonii, Theorem 6, gives two Demonstrations to prove, that Duple Ration contains, and is compos’d of the two next Rations, viz. Sesquialtera and Sesquitercia. Therefore an Octave which is in Duple Ration 2 to 1 is divided into, and compos’d of a Fifth, whose Ration is found to be Sesquialtera 3 to 2; and a Fourth, whose Ration is Sesquitercia 4 to 3. In like manner, Sesquialtera is compos’d of Sesquiquarta and Sesquiquinta; that is, a Fifth, 3 to 2, may be divided into a Third Major, 5 to 4; and a Third Minor, 6 to 5, &c.

There is an easie Way to take a view of the Mean Rations, which may be contain’d in any Ration given, by transferring the Prime or Radical Numbers of the given Ration into greater Numbers of the same Ration, as 2 to 1 into 4 to 2, or 6 to 3, &c.; which have the same Ration of Duple. Again, 3 to 2 into 6 to 4, which is still Sesquialtera. Now in 4 to 2 the Mediety is 3. So that 4 to 3, and 3 to 2, are comprehendent.
Of Conords.

ded in 4 to 2; that is, a Fourth and a Fifth are comprehended in an Eighth. In 6 to 4 the Mediety is 5, so 6 to 4 contains 6 to 5, and 5 to 4; i. e. a Fifth contains the two Thirds. Let 6 to 3 be the Octave, and it contains 6 to 5 Third less, 5 to 4 Third major, and 4 to 3 a Fourth, and hath two Medieties, 5 and 4. Of this I shall say more in the next Chapter.

These Rations express the Difference of Length in several Strings which make the Conords; and consequently the Difference of their Vibrations. Take two Strings A B, in all other respects equal, and compare their Lengths, which, if equal, make Unison, or the same Tune. If A be double in Length to B, i. e. 2 to 1, the Vibrations of B will be duple to those of A, and unite alternately, viz. at every Course, crossing at the Recourse, and give the Sound of an Octave to A.

If the Length of A be to that of B as 3 to 2, and consequently the Vibrations as 2 to 3, their Sounds will conform in a Fifth, and their Motions unite after every second Recourse, i. e. at every other or third Course.
Of Concords.

If A to B be as 4 to 3, they found a Fourth, their Motions uniting after every third Recourse, viz. at every fourth Course.

If A to B be as 5 to 4, they found a Ditone, or Third Major, and unite after every fourth Recourse, i.e. every fifth Course.

If A to B be as 6 to 5, they found a Trihemitone, or Third Minor, uniting after every fifth Recourse, at every sixth Course.

Thus, by the frequency of their being mix'd and united, the Harmony of joyn'd Concords is found so very sweet and pleasing; the Remoter being also combined by their relation to other Concords besides the Unison. The greater Sixth, 5 to 3, is within the compass of Rations between 1 and 6; but, I confess, the lesser Sixth, 8 to 5, is beyond it; but is the Complement of 6 to 5 to an Octave, and makes a better Concord by its Combinations with the Octave, and Fourth from the Unison; having the Relation of a Third Minor to One, and of a Third Major to the Other, and their Motions uniting accordingly. And the Sixth Major hath the same Advantage. Of these Combinations I shall have
Of Concord.

have occasion to say somewhat more, after I have made the subject in hand as plain as I can.

I propos'd the collating of two several strings, to express the comfort which is made by them; but otherwise, these rations are more certainly found upon the measures of a monochord, taken by being apply'd to the section of a canon, or a rule of the string's length divided into parts, as occasion requires; because there is no need so often to repeat *Ceteris paribus*, as is when several strings are collated. And if you take the rations as fractions, it will be more easie to measure out the given parts of a monochord, or single string extended on an instrument: Those parts of the string divided by a moveable bridge or fret put under, and made to sound; that sound, related to the sound of the whole, will give the interval sought after. *Ex. gr. * \( \frac{1}{2} \) of the chord gives an eighth, \( \frac{3}{7} \) give a fifth, \( \frac{4}{7} \) found a fourth, \( \frac{5}{7} \) found a third major, \( \frac{6}{7} \) a third minor, \( \frac{7}{7} \) a sixth major, \( \frac{8}{7} \) a sixth minor: Now we thus express these concords.
Of Conords.

Unison. 3d Min. 3d Maj. 4th 5th

6th Min. 6th Maj. 8th 3d & 5th. 4th & 6th.

I said, that all Conords are in Rations within the Number Six; and I may add, that all Rations within the Number Six are Conords: Of which take the following Scheme.

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<th>6 to 5</th>
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<td>3d Major</td>
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<td>5 to 4</td>
<td>3d Major</td>
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<td>17th Major</td>
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All that are Conords to the Unison, are also Conords to the Octave. And all that are Discords to the Unison, are Discords.
Of Concords.

cords to the Octave. And some of the Intermediate Concords, are Concords one to another; as, the two Thirds to the Fifth, and the Fourth to the two Sixths. So that the Unison, Third, Fifth, and Octave; or the Unison, Fourth, Sixth, and Octave, may be founded together to make a compleat Close of Harmony: I do not mean a Close to conclude with, for the Plagal is not such; but a compleat Close, as it includes all Concords within the compass of Diapason. A Scheme of which I have set down at the end of the foregoing Staff of five Lines, which containeth the Notes by which the aforesaid Concords are express'd. The former two, which ascend from the Unison, Gamut, by Third Major (or Minor) and Fifth, up to the Octave, are usually call'd Authentic, as relating principally to the Unison, and best satisfying the Ear to rest upon: The other two, which ascend by the Fourth and Sixth Minor (or Major) up to the same Octave, are call'd Plagal, as more combining with the Octave, seeming to require a more proper Bass Note, viz. an Eighth below the Fourth, and therefore not making a good concluding Close: And on the continual shifting these, or often changing them, depends the Variety of Harmony (as far as Consonancy reacheth, which is but as the Body of Musick) in all
Of Conords.

all Contrapunct chiefly, but indeed in all kinds of Composition. I do not exclude a Sprinkling of Discords, nor here meddle with Air, Measure, and Rythmus, which are the Soul and Spirit of Musick, and give it so great a commanding Power. The Plagal Moods descend by the same Intervals, by which the Authentick ascend; which is by Thirds and Fifths; and the Authentick descend the same by which the Plagal ascend, viz. by Fourths and Sixths; one chiefly relating to the Unison, the other to the Octave.

But that, for which I describ'd these full Closes, was chiefly to give (as I promis'd) a larger account of the before-mention'd Combinations of Conords, which encrease the Consonancies of each Note, and make a wonderful Variegation and Delightfulness of the Harmony.

Cast your Eye upon the first of them in the Authentick Scale, you will see that Bmi hath three Relations of Consonancy, viz. to the Unison, or given Note G; to the Fifth, and to the Octave: To the Unison as a Third minor; to the Fifth as a Third major; to the Octave a Sixth major; so that its Motions joyn after every fifth Recourse, i.e. at every sixth Course.
Of Conords.

with the Unison; every fifth with the Diapente or Fifth; every sixth Course with the Octave. Then consider the Diapente, D sol re; as a Fifth to the Unison, itjoyns with it every third Course; and as a Fourth to the Octave, they joyn every fourth Course. Then, the Octave with the Unison, joyns after every second Vibration, i.e. at every Course.

Now take a Review of the Variety of Conionancies in these four Notes. Here are mix’d together in one Confort the Rations of 2 to 1, 3 to 2, 4 to 3, 5 to 4, 6 to 5, 5 to 3. And just so it is in the other Clo- ses, only changing alternately the Sixths.

You may see here, within the space of three Intervals from the Unison, viz. 3d, 5th, and 8th, what a Concourse there is of Consonant Rations, to variegate and give (as ’twere) a pleasant Purling to the Harmony within that Space: For now, all this Variety is form’d within one System of Diapason, justly bearing that Name. But then, think what it will be when the remote Compounded Conords are joyn’d to them; as when we make a full Close with both Hands upon an Organ, or Harpsichord, or when the higher Part of a Confort of Music is reconcil’d to the lower, by the middle
Of Conords.

middle Parts, viz. the Treble to the Bafs, by the Mean and Tenor: And all this, refresh'd by the Interchangings made between the Plagal and Authentick Moods. Add to all this the infinite Variety of Movement of some Parts, thro' all Spaces, while some Part moves slowly; and (as in Fuges) one Part chafing and pursuing another.

The whole Reason of Consonancy being founded upon the Mixture and Uniting of the Vibrating Motions of several Chords, or founding Bodies, 'tis fit it should here be better explain'd and confirm'd. That their Mixtures accord to their Ratins, 'tis easie to be computed; but it may be represented to your Eye.
Of Conords.

\[\begin{align*}
V & \quad A \\
B & \quad V
\end{align*}\]

\[\begin{align*}
O & \quad A \\
B & \quad O
\end{align*}\]

\[\begin{align*}
D & \quad A \\
C & \quad B \\
B & \quad D
\end{align*}\]

<table>
<thead>
<tr>
<th></th>
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<th></th>
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<tbody>
<tr>
<td>O</td>
<td>AB BA AB BA</td>
<td>AB BA AB BA AB BA</td>
<td>AB BA</td>
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</table>

<table>
<thead>
<tr>
<th>V</th>
<th>A B</th>
<th>B A</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>ABC</td>
<td>CAB</td>
</tr>
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</table>
Let V V be a Chord, and stand for the Unison: Let O O be a Chord half so long, which will be an Octave to the Unison, and the Vibrations double: Then, I say, they will alternately (i.e. at every other Vibration) unite. Let from A to B be the Course of the Vibration, and from B to A the Recourse; observing by the way, that (in relation to the Figures mention'd in this Paragraph and the next, as also in the former Diagram of the Pendulum, cap. 2, pag. 9.) when I say, [from B to A] and [overtakes V in A, &c.] I do there endeavour to express the Matter brief and perspicuous, without perplexing the Figures with many Lines; and avoiding the Incumbrance of so many Cautions, whereby to distract the Reader: Yet I must always be understood to acknowledge the continual Decrease of the Range of Vibrations between A and B, while the Motion continues; and by A and B mean only the Extremities of the Range of all those Vibrations, both the First greatest, and also the Successive lessen'd, and gradually contracted Extremities of their Range. And the following Demonstration proceeds and holds equally in both, being apply'd to the Velocity of Recourses, and not to the Compass of their Range, which is not at all here consider'd. Such a kind of Equity, I must
must sometimes, in other parts of this Discourse, beg of the candid Reader. To proceed therefore, I say, whilst V being struck, makes his Course from A to B; O (struck likewise) will have his Course from A to B, and Recourse from B to A. Next, whilst V makes Recourse from B to A, O is making its Course contrary, from A to B, but recourseth, and overtakes V in A, and then they are united in A, and begin their Course together. So you see, that the Vibrations of Diapason unite alternately, joyning at every Course of the Unison, and crossing at the Recourse.

Thus also Diapente, or Fifth, having the Ration of 3 to 2, unites in like manner at every third Course of the Unison. Let the Chord D D be Diapente to the Unison V; whilst V courseth from A to B, the Chord D courseth from A to B, and makes half his Recourse as far as C; i.e. 3 to 2. Whilst V recourseth from B to A, D paffeth from C to A, and back from A to B. Whilst V courseth again from A to B, D paffeth from B to A, and back to C. Whilst V recourseth from B to A, D paffeth from C to B, and back to A; and then they unite in A, beginning their Courses together at every third Course of V. In like manner the rest of the Conords unite, at the
Of Conords.

The 4th, 5th, 6th Course, according to their Rations, as might this same way be shewn, but it would take up too much room, and is needless, being made evident enough from these Examples already given.

Thus far the Rates and Measures of Consonance lead us on, and give us the true and demonstrable Grounds of Harmony: But still 'tis not compleat without Discords and Degrees (of which I shall treat in another Chapter) intermix'd with the Conords, to give them a Foyl, and set them off the better. For (to use a homely Resemblance) that our Food, taken alone, tho' proper, and wholesome, and natural, may not cloy the Palate, and abate the Appetite, the Cook finds such kinds and varieties of Sawce, as quicken and please the Palate, and sharpen the Appetite, tho' not feed the Stomach; as Vinegar, Mustard, Pepper, &c. which nourish not, nor are taken alone, but carry down the Nourishment with better Relish, and assist it in Digestion. So the Practical Masters and Skilful Composers make use of Discords, judiciously taken, to relish the Confort, and make the Conords arrive much sweeter at the Ear, in all sorts of Descant, but most frequently in Cadence to a Close. In all which, the chief Regard is to be had to
to what the Ear may expect in the Conduct of the Composition, and must be performed with Moderation and Judgment; which I now only mention, not intending to treat of Composing, which is out of my Design and Sphere, and would be too large; but my Design is, to make these Grounds as plain as I can, thereby to gratifie those whose Philosophical Learning, without previous Skill in Musick, will easily render them capable of this Theory: And also those Masters in Practick Musick, and Lovers of it, who, tho' wanting Philosophy, and the Latin and other Foreign Tongues, to read better Authors; yet, by the help of their Knowledge in Musick, may attain to understand the depth of the Grounds and Reasons of Harmony, for whose sakes it is done in this Language.

I shall conclude this Chapter with some Remarks, concerning the Names given to the several Concords: We call them Third, Fourth, Fifth, Sixth, and Eighth. Of these the Thirds being Two, and Sixths being also Two, want better distinguishing Names. To call them Flat and Sharp Thirds, and Flat and Sharp Sixths, is not enough, and lies under a Mistake; I mean, it is not a sufficient Distinction, to call the greater Third
Third and Sixth, Sharp Third and Sharp Sixth, and the lesser, Flat. They are so indeed in ascending from the Unison, but in descending they are contrary; for to the Octave that greater Sixth is a lesser Third, and the greater Third is a lesser Sixth; which lesser Third and Sixth cannot well be call'd Flat, being in a Sharp Key; Flat and Sharp therefore do not well distinguish them in general; the lesser Third from the Octave being sharp, and the greater Sixth flat. So, from the Fifth descending by Thirds, if the first be a minor Third, it is sharp, and the other being a major Third, cannot be said to be flat.

The other Distinction of them, viz. by Major and Minor, is more proper, and does well express which of them we mean. But still the common and confused Name of Third, if the Distinction of major and minor be not always well remember'd, is apt to draw young Practitioners, who do not well consider, into another Error. I would therefore call the greater Third (as the Greeks do) Ditone, i.e. of two whole Tones; and the Third minor, Tribemitone or Sesquitone, as consisting of three half-Tones, (or rather of a Tone and half a Tone) and this would avoid the mention'd Error which I am going to describe.
It is a Rule in composing Consort Music, that it is not lawful to make a Movement of two Unisons, or two Eighths, or two Fifths together; nor of two Fourths, unless made good by the addition of Thirds in another Part: But we may move as many Thirds or Sixths together as we please. Which last is false, if we keep to the same sort of Thirds and Sixths; for the two Thirds differ one from another in like manner as the Fourth differs from the Fifth: For in the same manner as the Eighth is divided into a Fifth and Fourth, so is a Fifth into a Third major and Third minor. Now call them by their right Names, and, I say, it is not lawful to make a Movement of as many Ditones, or of as many Sesquitones as you please: And therefore when you take the Liberty spoken of, under the general Names of Thirds, it will be found that you mix Ditones and Tribemitones, and so are not concern’d in the aforesaid Rule; and so the Movements of Sixths will be made with mixture and interchanges of 6th major and 6th minor, which is safe enough.

Yet, I confess, there is a little more Liberty in moving Tribemitones and Ditones, as likewise either of the Sixths, than there is in moving Fourths or Fifths; and the
the Ear will bear it better. Nay, there is necessity, in a gradual Movement of Thirds, to make one Movement by two Tribemitone together in every Fourth and Fifth, or Fourth disjunct; that is, twice in Diapason, or, at least, in two Fifths; as in Gamut Key proper. The natural Ascent will be Ut Re Mi Fa Sol La: Now, to these join Thirds in Natural Ascent, and then they will be Mi Fa Sol La Fa Sol. {Mi Fa Sol La Fa Sol} {Ut Re Mi Fa Sol La} And thus it will be in other Cliffs, but with some variation, according to the Place of the Hemitone. Here {fa re} and {sol mi} are two Tribemitone succeeding one another, and you cannot well alter them without disor- dering the Ascent, and disturbing the Harmony; because, where there is a Hemitone, the Tone below join'd to it, makes a Tribemitone; and the next Tone above it, join'd to it, makes the same. Thus you see the necessity of moving two Tribemitone together, twice in Diapason, or a 9th, in progression of Thirds, in Diatonic Harmony, but you cannot well go fur- ther.

Now, there is Reason why two Tribemitone will better bear it, because of their different Relations, by which one Tribemitone
tone is better distinguish'd from another, than one Octave, or one Fifth, or one Fourth from another.

In a Third minor, which hath two Degrees or Intervals, consisting of a Tone and Hemitone, the Hemitone may be placed either in the lower Space, and then generally is united to his Third major (which makes the Complement of it to a Fifth) downward, and makes a sharp Key; or else it may be placed in the upper Space, and then generally takes his third major above, to make up the fifth upward, and constitute a flat Key. And thus a Tritone is avoided both ways. I say, if the Hemitone in the Third minor be below, then the Third major lies below it, and the Air is sharp. If the Hemitone be above, then the Third major lies above, and the Air is flat. And thus the two minor Thirds join'd in consequence of Movement, are differenc'd in their Relations, consequent to the place of the Hemitone; which Variety takes off all Nauseousness from the Movement, and renders it sweet and pleasant.

You cannot so well and regularly make a Movement of Ditones, tho' it may be done sometimes, once or twice, or more,
Of Conords.

in a Bearing Passage (in like manner as you may sometimes use Discords) to give, after a little grating, a better Relish. The Skilful Artist may go farther in the use of Thirds and Discords than is ordinarily allow'd.

I might enlarge this Chapter, by setting down Examples of the Lawful and Unlawful Movements of Thirds major and minor, and of the Use of Discords; but, as I said before, my Design is not to treat of Composition: However, you may cast your Eye upon these following Instances, and your own Observation from the best Masters will furnish you with the rest.

Lawful Movement of Thirds, Mix'd. Unlawful Movement of Thirds Major.

\[ \begin{align*}
&\text{Lawful Movement of Thirds, Mix'd.} \\
&\text{Unlawful Movement of Thirds Major.}
\end{align*} \]
Of Conords.

That the Reader may not incur any Mistake or Confusion, by several Names of the same Intervals, I have here set them down together, with their Rations.

<table>
<thead>
<tr>
<th>Intervals</th>
<th>Ratios</th>
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</thead>
<tbody>
<tr>
<td>8th</td>
<td>Octave, Diapason.</td>
</tr>
<tr>
<td>9th Major</td>
<td>Heptachord Major.</td>
</tr>
<tr>
<td>7th Minor</td>
<td>Heptachord Minor.</td>
</tr>
<tr>
<td>6th Major</td>
<td>Hexachord Major.</td>
</tr>
<tr>
<td>6th Minor</td>
<td>Hexachord Minor.</td>
</tr>
<tr>
<td>5th</td>
<td>Diapente, Pentachord.</td>
</tr>
<tr>
<td>5th False (in defect)</td>
<td>Semidiapente.</td>
</tr>
<tr>
<td>4th False (in excess)</td>
<td>Tritone.</td>
</tr>
<tr>
<td>3d Major</td>
<td>Diatessaron, Tetrachord.</td>
</tr>
<tr>
<td>3d Minor</td>
<td>Ditone.</td>
</tr>
<tr>
<td>2d Major or Whole Note Major</td>
<td>Tone Major</td>
</tr>
<tr>
<td>2d Minor or Whole Note Minor</td>
<td>Tone Minor</td>
</tr>
<tr>
<td>2d Least, or Half Note Greater</td>
<td>Hemi-tone Semi-tone Major</td>
</tr>
<tr>
<td>Half-Note Least</td>
<td>Hemi-tone Semi-tone Minor</td>
</tr>
<tr>
<td>Quarter-Note</td>
<td>Diatessis Chromatic.</td>
</tr>
<tr>
<td>Difference between Tone Major, and Tone Minor</td>
<td>Comma. Comma Majus.</td>
</tr>
</tbody>
</table>

Note, Whenever I mention Diatessis without distinction, I mean Diatessis Minor, or Enharmonic: And when I set mention Comma, I mean Comma Majus, or Schism.
Of Conords.

I should next treat of Dis cords, but because there will intervene so much Use of Calculation, it is needful that (before I go further) I premise some account of Proportion in General, and apply it to Harmony.

CHAP. V.

Of Proportion; and apply'd to Harmony.

WHEREAS it hath been said before, That Harmonick Bodies and Motions fall under Numerical Calculations, and the Rations of Conords have been already assign'd; it may seem necessary here (before we proceed to speak of Dif cords) to shew the Manner how to calculate the Proportions appertaining to Harmonick Sounds: And for this I shall better prepare the Reader, by premising something concerning Proportion in General.

We may compare (i. e. amongst themselves) either (1.) Magnitudes, (so they be of the same kind;) or (2.) the Gravitations, Motions, Velocities, Duration, Sounds, &c. from thence arising; or fur-
ther, if you please, the Numbers themselves, by which the Things compar'd are explicated. And if these shall be unequal, we may then consider, either, First, how much one of them exceeds the other; or, Secondly, after what manner one of them stands related to the other, as to the Quotient of the Antecedent (or former Term) divided by the Consequent (or latter Term): Which Quotient doth expound, denominate, or shew, how many times, or how much of a time or times, one of them doth contain the other. And this by the Greeks is call'd ἀναλογία, Ratio; as they are wont to call the Similitude, or Equality of Ratio's ἀναλογία, Analogie, Proportion, or Proportionality: But Custom, and the Sense assisting, will render any over-curious Application of these Terms unnecessary.

From these two Considerations last mention'd there are wont to be deduced three sorts of Proportion, Arithmetical, Geometrical, and a mix'd Proportion resulting from these two, call'd Harmonical.

1. *Arithmetical*, when three or more Numbers in Progression have the same Difference; as, 2, 4, 6, 8, &c. or discontinued, as 2, 4, 6; 14, 16, 18.
2. Geometrical, when three or more Numbers have the same Ration, as \(2, 4, 8, 16, 32\); or discontinued, \(2, 4, 64, 128\).

Lastly, Harmonical, (partaking of both the other) when three Numbers are so order'd that there be the same Ration of the Greatest to the Least, as there is of the Difference of the two Greater to the Difference of the two Less Numbers: As in these three Terms, \(3, 4, 6\), the Ration of 6 to 3 (being the greatest and least Terms) is Duple. So is 2, the Difference of 6 and 4 (the two greater Numbers) to 1, the Difference of 4 and 3 (the two less Numbers) Duple also. This is Proportion Harmonical, which Diapason, 6 to 3, bears to Diapente 6 to 4, and Diatessaron 4 to 3, as its mean Proportionals.

Now for the Kinds of Rations most properly so call'd, i.e. Geometrical: First observe, that in all Rations the former Term or Number (whether greater or less) is always call'd the Antecedent, and the other following Number is call'd the Consequent. If therefore the Antecedent be the greater Term, then the Ration is either Multiplex, Superparticular, Superpartient, or (what is compounded of these) Mutix-
plex Superparticular, or Multiplex Superpartient.

1. Multiplex; as Duple, 4 to 2; Triple, 6 to 2; Quadruple, 8 to 2.

2. Superparticular; as, 3 to 2, 4 to 3, 5 to 4, exceeding but by one aliquot part, and in their Radical or least Numbers, always but by one; and these Rations are term'd Sesquialtera, Sesquitertia (or Supertertia) Sesquiquarta, (or Superquarta) &c.

Note, that Numbers exceeding more than by one, and but by one aliquot part, may yet be Superparticular, if they be not express'd in their Radical, i.e. least Numbers; as 12 to 8 hath the same Ration as 3 to 2; i.e. Superparticular, tho' it seem not so till it be reduced by the greatest Common Divisor to its Radical Numbers 3 to 2. And the Common Divisor (i.e. the Number by which both the Terms may severally be divided) is often the Difference between the two Numbers; as in 12 to 8, the Difference is 4, which is the Common Divisor. Divide 12 by 4, the Quotient is 3; divide 8 by 4, the Quotient is 2; so the Radical is 3 to 2. Thus also 15 to 10 divided by the Difference 5, gives 3 to 2; yet, in 16 to 10, 2 is the Common Divisor, and gives 8 to 5, being Superpartient.
tient. But in all Superparticular Rations, whose Terms are thus made larger by being multiply'd, the Difference between the Terms is always the greatest Common Divisor; as in the foregoing Examples.

The third kind of Ration is Superpartient, exceeding by more than One, as 5 to 3, which is call'd Superbipartiens Tertias (or Tria) containing 3 and \( \frac{2}{3} \); 8 to 5, Supertripartiens Quintas, 5 aud \( \frac{3}{5} \).

The fourth is Multiplex Superparticular, as 9 to 4, which is duple, and Sesquiquartia, 13 to 4, which is triple, and Sesquiquarta.

The fifth and last is Multiplex Superpartient, as 11 to 4; duple, and Supertripartiens Quartas.

When the Antecedent is less than the Consequent, viz. when a less is compar'd to a greater, then the same Terms serve to express the Rations, only prefixing Sub to them; as, Submultiplex, Subsuperparticular (or Subparticular) Subsuperpartient (or Subpartient) &c. 4 to 2 is Duple, 2 to 4 is Subduple. 4 to 3 is Sesquitertia; 3 to 4 is Subsesquitertia; 5 to 3 is Superbipartiens Tertias; 3 to 5 is Subsuperbipartiens Tertias, &c.
This short Account of Proportion was necessary, because almost all the Philosophy of Harmony consists in Rations, of the Bodies, of the Motions, and of the Intervals of Sound, by which Harmony is made.

And in searching, stating, and comparing the Rations of these, there is found so much Variety, and Certainty, and Facility of Calculation, that the Contemplation of them may seem not much less delightful than the very Hearing the good Musick it self, which springs from this Fountain. And those who have already an affection for Musick cannot but find it improv'd and much enhaunc'd by this pleasant recreating Chace (as I may call it) in the large Field of Harmonic Rations and Proportions, where they will find, to their great Pleasure and Satisfaction, the hidden Causes of Harmony (hidden to most, even to Practitioners themselves) so amply discover'd and laid plain before them.

All the Habitudes of Rations to each other are found by Multiplication or Division of their Terms; by which any Ration is added to, or subtracted from another; And there may be use of Progression of Ra-
Of Proportion.

Rations, or Proportions, and of finding a Medium or Mediety between the Terms of any Ration: But the main Work is done by Addition and Subtraction of Rations; which, tho' they are not perform'd like Addition and Subtraction of Simple Numbers in Arithmetick, but upon Algebraick Grounds, yet the Praxis is most easie.

One Ration is added to another Ration, by multiplying the two antecedent Terms together; i.e. the Antecedent of one of the Rations by the Antecedent of the other (for the more ease they should be reduced into their least Numbers or Terms) and then the two Consequent Terms in like manner. The Ration of the Product of the Antecedents, to that of the Product of the Consequents, is equal to the other two added or join'd together. Thus (for Example) add the Ration of 8 to 6; i.e. (in Radical Numbers) 4 to 3, to the Ration of 12 to 10; i.e. 6 to 5, the Product will be 24 and 15; i.e. 8 to 5. You may set 'em thus, and multiply 4 by 6, they make 24, which set at the bottom; then multiply 3 by 5, they make 15, which likewise set under, and you have 24 to 15; which is a Ration compounded of
of the other two, and equal to them both. Reduce these Products, 24 and 15, to their least Radical Numbers, which is, by dividing as far as you can find a common Divisor to them both (which is here done by 3) and that brings them to the Ration of 8 to 5. By this you see, that a Third minor, 6 to 5, added to a Fourth, 4 to 3, makes a Sixth minor, 8 to 5. If more Rations are to be added, set them all under each other, and multiply the first Antecedent by the second, and that Product by the third, and again that Product by the Fourth, and so on; and so in like manner the Consequents.

This Operation depends upon the Fifth Proposition of the Eighth Book of Euclid; where he shews, that the Ration of Plain Numbers is compounded of their Sides. See these Diagrams:

Now
Now compound these Sides. Take for the Antecedents, 4 the greater Side of the greater Plane, and 3 the greater Side of the less Plane, and they multiply'd give 12: Then take the remaining two Numbers 3 and 2, being the less Sides of the Planes (for Consequents) and they give 6. So the Sides of 4 and 3, and of 3 and 2, compounded (by multiplying the Antecedent Terms by themselves, and the Consequents by themselves) make 12 to 6, i.e. 2 to 1; which being apply'd, amounts to this; \( \text{Ratio Sesquialtera, } 3 \text{ to } 2 \), added to \( \text{Ratio Sesquitertia } 4 \text{ to } 3 \), makes Duple Ration, 2 to 1. Therefore Diapente added to Diateffaron, makes Diapason.

**Subtraction** of one Ration from another greater is perform'd in like manner by multiplying the Terms; but this is done not *Laterally*, as in Addition, but *Crosswise*; by multiplying the Antecedent of the former (i.e. of the greater) by the Consequent of the latter, which produceth a new Antecedent; and the Consequent of the former by the Antecedent of the latter, which gives a new Consequent. And therefore it is usually done by an Oblique Decussation of the Lines. For Example, If you would take 6 to 5 out of 4 to 3, you
you may set them down as in the Margin; Then 4 multiply'd by 5,
4 - 3
X
6 - 5
4 - 3 makes 20, and 3 by 6 gives 18:
So 20 to 18, i.e. 10 to 9, is the Remainder. That is, subtract a
Third Minor out of a Fourth,
20 - 18, and there will remain a Tone

Multiplication of Rations is the same with their Addition, only 'tis not wont
to be of divers Rations, but of the same, being taken twice, thrice, or oftener, as you please. And as before in Addition you added
divers Rations by multiplying them, so here in Multiplication you add the same Ration
to itself, after the same manner, viz. by
multiplying the Terms of the same Ration
by themselves; i.e. the Antecedent by itself, and the Consequent by itself, (which
in other Words is to multiply the same by 2) and will, in the Operation, be to square
the Ration first propounded (or give the
Second Ordinal Power, the Ration first gi-
gen being the First Power or Side.) And
to this Product, if the Simple Ration shall
again be added (after the same manner as
before) the Aggregate will be triple of the
Ration first given; or the Product of that
Ration multiply'd by 3, viz. the Cube, or
Third Ordinal Power. Its Biquadrate, or
Fourth
Fourth Power, proceeds from multiplying it by 4, and so successively in order as far as you please you may advance the Powers. For instance, the Duple Ration, 2 to 1, being added to it self, dupled, or multiply’d by 2, produceth 4 to 1, (the Ration Quadruple): And if to this, the first again be added, which is equivalent to multiplying that said first by 3) there will arise the Ration Octuple, or 8 to 1. Whence the Ration 2 to 1 being taken for a Root, its Duple 4 to 1 will be the Square, its Triple 8 to 1 the Cube thereof, &c. as hath been said above. And, to use another Instance, To duple the Ration of 3 to 2, it must be thus squar’d; 3 by 3 gives 9: 2 by 2 gives 4: so the Duple or Square of 3 to 2 is 9 to 4. Again, 9 by 3 is 27, and 4 by 2 is 8, so the Cubic Ration of 3 to 2 is 27 to 8. Again, to find the Fourth Power, or Biquadrate, (i.e. squar’d Square) 27 by 3 is 81, 8 by 2 is 16; so 81 to 16 is the Ration of 3 to 2 quadrupled, as ’tis dupled by the Square, tripled by the Cube, &c. To apply this Instance to our present purpose, 3 to 2 is the Ration of Diapente, or a Fifth in Harmony; 9 to 4 is the Ration of twice Diapente, or a Ninth (viz. Diapason with Tone Major); 27 to 8 is the Ration of thrice Diapente, or three Fifths, which is Diapason with Six Major (viz. 13th Major)
The Ration of 81 to 16 makes four Fifths, i.e. Dis-diapason, with two Tones Major, i.e. a Seventeenth Major, and a Comma of 81 to 80.

To divide any Ration, you must take the contrary Way, and by extracting of these Roots respectively, Division by their Indices will be perform'd. E.gr. To divide it by 2, is to take the Square Root of it; by 3, the Cubic Root; by 4, the Biquadratick, &c. Thus to divide 9 to 4 by 2, the Square Root of 9 is 3, the Square Root of 4 is 2; then 3 to 2 is a Ration just half so much as 9 to 4.

From hence it will be obvious to any to make this Inference; That Addition and Multiplication of Rations are (in this case) one and the same thing. And these Hints will be sufficient to such as bend their Thoughts to these kinds of Speculations, and no great Trespass upon those that do not.

The Advantage of proceeding by the Ordinal Powers, Square, Cube, &c. (as is before mention'd) may be very useful where there is Occasion of large Progressions; as, to find (for Example) how many Comma's are contain'd in a Tone Major, or other Interval.
Of Proportion.

Let it be, How many are in Diapason? Which must be done by multiplying Comma's, i.e. adding them, till you arrive at a Ration equal to Octave, (if that be sought) viz. Duple: Or else by dividing the Ration of Diapason by that of a Comma, and finding the Quotient; which may be done by Logarithms. And herein I meet with some Differences of Calculations.

Mersennus finds, by his Calculation, 58 1/4 Comma's, and somewhat more, in an Octave: But the late Nicholas Mercator, a Modest Person, and a Learned and Judicious Mathematician, in a Manuscript of his, of which I have had a Sight, makes this Remark upon it; In solvendo hoc Problemate aberrat Mersennus: And he, working by the Logarithms, finds out but 55, and a little more; and from thence has deduced an ingenious Invention of finding and applying a least Common Measure to all Harmonic Intervals, not precisely perfect, but very near it.

Supposing a Comma to be 1/17 part of Diapason; for better Accommodation rather than according to the true Partition 1/17, which 1/17 he calls an Artificial Comma, not exact, but differing from the true Natural
tural Comma about $\frac{1}{4}$ part of a Comma, and $\frac{1}{4}$ of Diapason (which is a Difference imperceptible) then the Intervals within Diapason will be measur'd by Comma's according to the following Table; which you may prove by adding two, or three, or more of these Numbers of Comma's, to see how they agree to constitute those Intervals, which they ought to make; and the like by subtracting.

<table>
<thead>
<tr>
<th>Intervals</th>
<th>$\frac{1}{4}$</th>
<th>Intervals</th>
<th>$\frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comma</td>
<td>1</td>
<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
<td>22</td>
</tr>
<tr>
<td>Diecis</td>
<td>2</td>
<td>Tritone</td>
<td>26</td>
</tr>
<tr>
<td>Semit. Minus</td>
<td>3</td>
<td>Semidiapente</td>
<td>27</td>
</tr>
<tr>
<td>Semit. Medium</td>
<td>4</td>
<td>5&lt;sup&gt;th&lt;/sup&gt;</td>
<td>31</td>
</tr>
<tr>
<td>Semit. Majus</td>
<td>5</td>
<td>6&lt;sup&gt;th&lt;/sup&gt; Minor &amp; 36</td>
<td></td>
</tr>
<tr>
<td>Semit. Maximus</td>
<td>6</td>
<td>6&lt;sup&gt;th&lt;/sup&gt; Major</td>
<td>39</td>
</tr>
<tr>
<td>Tone Minor</td>
<td>8</td>
<td>7&lt;sup&gt;th&lt;/sup&gt; Minor</td>
<td>45</td>
</tr>
<tr>
<td>Tone Major</td>
<td>9</td>
<td>7&lt;sup&gt;th&lt;/sup&gt; Major</td>
<td>48</td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt; Minor</td>
<td>14</td>
<td>Octave</td>
<td>53</td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt; Major</td>
<td>17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This I thought fit, on this Occasion, to impart to the Reader, having Leave so to do from Mr. Mercator's Friend, to whom he presented the said Manuscript.

Here I may advertise the Reader, that it is indifferent whether you compare the greater
greater Term of an Harmonic Ration to the less, or the less to the greater; i.e. whether of them you place as Antecedent, e.g. 3 to 2, or 2 to 3; because in Harmonies the Proportions of Lengths of Chords, and of their Vibrations, are reciprocal or counter-chang'd: As the Length is encreas'd, so the Vibrations are in the same proportion decreas'd; &c. è contra.

If therefore (as in Diapente) the length of the Unison String be 3, then the length (cateris paribus) of the String which in ascent makes Diapente to that Unison must be 2, or \( \frac{2}{3} \): Thus the Ration of Diapente is 2 to 3 in respect of the Length of it, compar'd to the Length of the Unison String.

Again, the String 2 vibrates thrice in the same Time that the String 3 vibrates twice; and thus the Ration of Diapente, in respect of Vibrations, is 3 to 2: So that where you find in Authors sometimes the greater Number in the Rations set before and made the Antecedent, sometimes set after and made the Consequent, you must understand in the former, the Ration of their Vibrations; and in the latter, the Ration of their Lengths; which comes all to one.

G 2

or,
Or, you may understand the Unison to be compar’d to Diapente above it, and the Ration of Lengths is 3 to 2, of Vibrations 2 to 3, or else Diapente to be compar’d to the Unison, and then the Ration of Lengths is 2 to 3, of Vibrations 3 to 2. This is true in single Rations, or if one Ration be compar’d to another; then the two greater Terms must be rank’d as Antecedents; or otherwise, the two lesser Terms.

The Difference between Arithmetical and Geometrical Proportion is to be well heeded. An Arithmetical mean Proportion is that which has equal Difference to the Antecedent and Consequent Terms of those Numbers to which it is the Mediety, and is found by adding the Terms, and taking half the Sum. Thus between 9 and 1, which added together make 10, the Mediety is 5; being Equidifferent from 9 and from 1; which Difference is 4: As 5 exceeds 1 by 4; so likewise 9 exceeds 5 by 4. And thus in Arithmetical Progression 2, 4, 6, 8; where the Difference is only consider’d, there is the same Arithmetical Proportion between 2 and 4, 4 and 6, 6 and 8; and between 2 and 6, and 4 and 8: But in Geometrical Proportion, where is consider’d not the Numerical Difference, but another Habitude of the Terms, viz. how many times,
times, or how much of a time or times, one of them doth contain the other (as hath been explain'd at large in the beginning of this Chapter.) There the Mean Proportional is not the same with Arithmetical, but found another way; and equidifferent Proportions make different Rations. The Rations (taking them all in their least Terms) express'd by less Numbers, being greater than those of greater Numbers, I mean in Proportions super particular, &c. where the Antecedents are greater than the Consequents, (as, on the contrary, where the Antecedents are less than the Consequents, the Ratio's of less Numbers are less than the Ratio's of greater.) The Mediety of 9 to 1 is not now 5, but 3; 3 having the same Ration to 1 as 9 has to 3, (as 9 to 3, so 3 to 1) viz. triple. And so in Progression Arithmetical, of Terms having the same Differences; if consider'd Geometrically, the Terms will all be comprehended by unequal Rations. The Differences of 2 to 4, 4 to 6, 6 to 8, are equal, but the Rations are unequal; 2 to 4 is less than 4 to 6, and 4 to 6 less than 6 to 8. As on the contrary, 4 to 2 is greater than 6 to 4, and 6 to 4 greater than 8 to 6: For 4 to 2 is duple, 6 to 4 but Sesquialtera (one and a half only, or ½) and 8 to 6 is no more than Sesquitertia, (one and a third part,
part, or \( \frac{3}{4} \) which shews a considerable inequality of their Rations. In like manner 6 to 2 is triple; 8 to 4 is but duple, and yet their Differences equal. Thus the mean Rations comprehended in any greater Ration divided Arithmetically, \( i.e. \) by equal Differences, are unequal to one another, consider'd Geometrically. Thus 2, 3, 4, 5, 6, if you consider the Numbers, make an Arithmetical Progression: But if you consider the Rations of those Numbers, as is done in Harmony, then they are unequal, every one being greater or less (according as you proceed by Ascent or Descent) than the next to it. Thus, in this Progression, (understanding, together with the Ratio's, the Intervals themselves, as is before premised) 2 to 3 is the greatest, being \( Diapente \); 3 to 4 the next, \( Diatessaron \); 4 to 5 still less, \( viz. \) \( Ditone \); 5 to 6 the least, being Sesquitone. Or, if you descend, 6 to 5 least; 5 to 4 next, \( \&c. \) These are the mean Rations comprehended in the Ration of 6 to 2, by which \( Diapason \) \( cum \) \( Diapente \), or a 12th, is divided into the aforesaid Intervals, and measured by them, \( viz. \) as is 6 to 2, \( \{ viz. \) triple \) so is the Aggregate of all the mean Rations within that Number, 6 to 5, 5 to 4, 4 to 3, and 3 to 2: Or 6 to 5, 5 to 2; or 6 to 4, 4 to 2; or 6 to 3, 3 to 2. The Aggregates of these are equal to 6 to 2, \( viz. \) triple. This
This is premised in order to proceed to what was intimated in the foregoing Chapter.

Taking notice first of this Procedure, peculiar to Harmonics, viz. to make Progression or Division in Arithmetical Proportion in respect of the Numbers; but to consider the things number'd according to their Rations Geometrical. And thus Harmonic Proportion is said to be compounded of Arithmetical and Geometrical.

You may find them all in the Division of the System of Diapason into Diapente and Diatessaron, i.e. 5th and 4th, ascending from the Unison.

If by Diapente first, then by 2, 3, 4, Arithmetically. If first by Diatessaron, then by 3, 4, 6, Harmonically. And these Rations consider'd Geometrically, in relation to Sound, there is likewise found Geometrical Proportions between the Numbers 6, 3 to 4, 2; and 6, 4 to 3, 2.

The Ancients therefore owning only 8th, 5th, and 4th, for simple Consonant Intervals, concluded them all within the Numbers of 12, 9, 8, 6, which contain'd them all,
Of Proportion.

all: viz. 12 to 6, Diapason; 12 to 8, Diapente; 12 to 9, Diatessaron; 9 to 8, Tone. And which serv'd to express the three kinds of Proportion, viz. Harmonical, between 12 to 8, and 8 to 6; Arithmetical, between 12 to 9, and 9 to 6; and Geometrical, between 12 to 9 and 8 to 6; and between 12 to 8, and 9 to 6. It was said therefore, that Mercurius's Lyre was strung with four Chords, having those Proportions, 6, 8, 9, 12.

Gassendi.

I intimated, that I would here more largely explain that ready and easie Way of finding and measuring the mean Rations contain'd in any of those Harmonic Rations given, by transferring them out of their Prime or Radical Numbers into greater Numbers of the same Ration. By dupling (not the Ration, but the Terms of it; still continuing the same Ration) you will have one Mediety; as, 2 to 1 dupled is 4 to 2; and you have 3 the Mediety. By tripling you will have two Means; 2 to 1 tripled is 6 to 3, containing 3 Rations; 6 to 5, 5 to 4, 4 to 3; and to still more, the more you multiply it.

Now observe, first, that any Ration Multiplex or Superpartient (or by transferring it out of its Radical Numbers made like
like Superpartient) contains so many Super-
particular Rations, as there are Units in
the Difference between the Antecedent
and the Consequent. Thus in 8 to 4
(being 2 to 1 transferr'd by quadrupling)
the Difference is 4, and it contains 4 Su-
perparticular Rations, viz. 8 to 7, 7 to 6,
6 to 5, and 5 to 4; where tho' the Pro-
gression of Numbers is Arithmetical, yet
the Proportions of Excess are Geometrical
and Unequal. The Superparticular Ra-
tions express'd by less Numbers being grea-
ter (as hath been said) than those that con-
sist of greater Numbers; 5 to 4 is a greater
Ration than 6 to 5, and 6 to 5 greater
than 7 to 6, and 7 to 6 than 8 to 7; as a
Fourth part is greater than a Fifth, and a
Fifth greater than a Sixth, &c. But in
this Instance there are two Rations not ap-
pertaining to Harmonics, viz. 8 to 7, and
7 to 6.

Secondly therefore, you may make un-
equal Steps, and take none but Harmonic
Rations, by selecting greater and fewer
intermediate Rations, tho' some of them
compos'd of several Superparticulars; pro-
vided you do not discontinue the Rational
Progression, but that you repeat still the
last Consequent, making it the next Anite-
cedent; as if you measure the Ration of
8 to 4.
8 to 4, by 8 to 6 and 6 to 4, or by 8 to 5 and 5 to 4, or by 8 to 6, and 6 to 5, and 5 to 4; in these three ways the Rations are all Harmonical, and are respectively contain'd in, and make up the Ration of 8 to 4. Thus you may measure, and divide, and compound most harmonic Rations without your Pen.

To that End I would have my Reader to be very perfect in the Radical Numbers which express the Rations of the seven first (or uncompounded) Consonants, viz. Diapason, 2 to 1; Diapente, 3 to 2; Diatessaron, 4 to 3; Ditone, 5 to 4; Trihemitone, 6 to 5; Hexachordon Majus, 5 to 3; Hexachordon Minus, 8 to 5; and likewise of the Degrees in Diatonick Harmony, viz. Tone Major, 9 to 8; Tone Minor, 10 to 9; Hemitone Major, 16 to 15; and the Differences of those Degrees; Hemitone Greatest, 27 to 25; and Hemitone Minor, 25 to 24; Comma, or Schism, 81 to 80; Diesis Enharmonic, 128 to 125.

Of other Hemitones I shall treat in the Eighth Chapter.

Now if you would divide any of the Consonants into two Parts, you may do it by the Mean or Mediety of the two Radical
cal Numbers, if they have a Mean; and where they have not, (as when their Ratio's are Superparticular) you need but duple those Numbers, and you will have a Mean (one or more.) Thus duple the Numbers of the Ration of Diapason, 2 to 1, and you have 4 to 2; and then 3 is the Mean by which it is divided into two unequal, but proper and harmonical parts, viz. 4 to 3, and 3 to 2. After this manner Diapason, 4 to 2, comprehends 4 to 3, and 3 to 2; So Diapente, 6 to 4, is 6 to 5, and 5 to 4: Ditone, 10 to 8, is 10 to 9, and 9 to 8; so Sixth major, 5 to 3, is 5 to 4, and 4 to 3.

Tho', from what was now observ'd, you may divide any of the Consonants into intermediate Parts, yet when you divide these three following, viz. Sixth minor, Diatessaron, and Trihemitone, you will find that those Parts into which they are divided, are not all such Intervals as are harmonical. The Sixth minor, whose Ration is 8 to 5, contains in it three Means, viz. 8 to 7, 7 to 6, and 6 to 5; the last whereof only is one of the harmonick Intervals, of which the Sixth minor consists, viz. Trihemitone; and to make up the other Interval, viz. Diatessaron, you must take the other two, 8 to 7, and 7 to 6; which being
Of Proportion.

ing added (or, which is the same thing, taking the Ratio of their two Extream Terms, that being the Sum of all the intermediate ones added) you have 8 to 6, or (in the least Terms) 4 to 3. Again, Diatesaron, in Radical Numbers 4 to 3; being (if those Numbers are dupled) 8 to 6, gives for his Parts 8 to 7, and 7 to 6; which Ratio's agree with no Intervals that are Harmonick; therefore you must take the Ration of Diatesaron in other Terms, which may afford such Harmonick Parts. And to do this, you must proceed farther than dupling (or adding it once to itself) for to this Duple you must add the first Radical Numbers once again (which in effect is the same with tripling it at first) viz. 4 and 3, to 8 and 6; and the Aggregate will be a new, but equivalent, Ration of Diatesaron; viz. 12 to 9. And this gives you three Means, 12 to 11, and 11 to 10; both Unharmonical; but which together are, as was shew'd before, the same with 12 to 10 (or 6 to 5) Tribemitone; and 10 to 9 Tone minor; and are the two Harmonical Intervals of which Diatesaron consists, and which divide it into the two nearest equal Harmonick Parts. Lastly, Tribemitone, or Third minor, 6 to 5; or (those Numbers being dupled) 12 to 10, gives 12 to 11, and 11 to 10, which are Un-
Of Proportion.

Unharmonical Rations; but tripled (after the former manner) 6 to 5 gives 18 to 15, which divides itself (as before) into 18 to 16, Tone major; and 16 to 15, Hemitone major.

Thus, by a little Practice, all Harmonick Intervals will be most easily measur’d, by the lesser Intervals compriz’d in them. Now, for Exercise sake, take the Measures of a greater Ration: Suppose that of 16 to 3 be given as an Harmonick System. To find what it is, and of what Parts it consists; first find the gross Parts, and then the more minute. You will presently judge, that 16 to 8 (being a Part of this Ration) is Diapason; and 8 to 4 is likewise Diapason. Then 16 to 4 is Disdiapason, or a Fifteenth, and the remaining 4 to 3 is a Fourth. So then 16 to 3 is Disdiapason and Diatessaron; i.e. an Eighteenth; 16 to 8, 8 to 4, and 4 to 3.
Of Proportion.

But, to find all the Harmonick Intervals within that Ration (for we now consider Rations as relating to Harmony) take this Scheme.

16 to 3 contains.

In Radicals.

<table>
<thead>
<tr>
<th>16 to 15,</th>
<th>15 to 12,</th>
<th>12 to 10,</th>
<th>10 to 9,</th>
<th>9 to 8,</th>
<th>8 to 6,</th>
<th>6 to 5,</th>
<th>5 to 4,</th>
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<tr>
<td>5 to 4,</td>
<td>6 to 5,</td>
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<td>4 to 3,</td>
<td>4 to 3,</td>
<td>4 to 3,</td>
<td>4 to 3,</td>
<td>4 to 3,</td>
</tr>
</tbody>
</table>

Tot. 16 to 3 | Disdiapason cum Diantessaron.

Or thus,

In Radicals.

<table>
<thead>
<tr>
<th>16 to 10,</th>
<th>10 to 6,</th>
<th>6 to 4,</th>
<th>4 to 3,</th>
</tr>
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<tbody>
<tr>
<td>8 to 5,</td>
<td>5 to 3,</td>
<td>3 to 2,</td>
<td>6th Minor.</td>
</tr>
<tr>
<td>6th Major.</td>
<td>5th</td>
<td>4th</td>
<td></td>
</tr>
</tbody>
</table>

Tot. 16 to 3 | Eighteenth.
Of Proportion.

All these Intervals thus put together are comprehended in, and make up, the Ration of 16 to 3, being taken in a conjunct Series of Rations.

But otherwise, within this compass of Numbers are contain’d many more Expressions of Harmonick Ration. Ex. gr.

<table>
<thead>
<tr>
<th>Radicals</th>
<th>Radicals</th>
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<tbody>
<tr>
<td>16 to 15,</td>
<td>12 to 6, 2 to 1.</td>
</tr>
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<td>16 to 12, 4 to 3.</td>
<td>12 to 4, 3 to 1.</td>
</tr>
<tr>
<td>16 to 10, 8 to 5.</td>
<td>12 to 3, 4 to 1.</td>
</tr>
<tr>
<td>16 to 8, 2 to 1.</td>
<td>10 to 9,</td>
</tr>
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<td>16 to 6, 8 to 3.</td>
<td>10 to 8, 5 to 4.</td>
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<td>16 to 4, 4 to 1.</td>
<td>10 to 6, 5 to 4.</td>
</tr>
<tr>
<td>16 to 3.</td>
<td>10 to 5, 2 to 1.</td>
</tr>
<tr>
<td>15 to 12, 5 to 4.</td>
<td>9 to 8,</td>
</tr>
<tr>
<td>15 to 10, 3 to 2.</td>
<td>9 to 6, 3 to 2.</td>
</tr>
<tr>
<td>15 to 5, 3 to 1.</td>
<td>9 to 3, 3 to 1.</td>
</tr>
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<td>15 to 3, 5 to 1.</td>
<td>8 to 6, 4 to 3.</td>
</tr>
<tr>
<td>14 to 7, 2 to 1.</td>
<td>8 to 5,</td>
</tr>
<tr>
<td>12 to 10, 6 to 5.</td>
<td>8 to 4, 2 to 1.</td>
</tr>
<tr>
<td>12 to 9, 4 to 3.</td>
<td>6 to 5, &amp;c.</td>
</tr>
<tr>
<td>12 to 8, 3 to 2.</td>
<td>Vid. Pag. 67.</td>
</tr>
</tbody>
</table>

And now I suppose the Reader better prepar’d to proceed in the remainder of this Discourse, where we shall treat of Discords.

CHAP.
Chap. VI.
Of Discords and Degrees.

All Habitudes of one Chord to another, that are not Concorrds, (such as are before describ’d) are Discords; which are or may be innumerable, as are the minute Tensions by which one Chord may be made to vary from itself, or from another. But here we are to consider only such Discords as are useful (and in truth necessary) to Harmony, or at least relating to it, as are the Differences found between Harmonick Intervals.

And these apt and useful Discords are either simple uncompounded Intervals, such as immediately follow one another, ascending or descending in the Scale of Musick; as, Ut, Re, Mi, Fa, Sol, La, Fa, Sol, and are call’d Degrees: Or else greater Spaces or Intervals compounded of Degrees including or skipping over some of them, as all the Concorrds do, Ut Mi, Ut Fa, Ut Sol, &c. And such are the Discords of which we now
now treat, as principally the *Tritone*, False Fifth, and the two Sevenths, Major and Minor, if they be not rather among the Degrees, &c. For more Perspicuity, I shall treat of them severally, viz. of Degrees, of Discords, and of Differences.

And first of Degrees.

Degrees are uncompounded Intervals, which are found upon eight Chords, and in seven Spaces, by which an immediate Ascent or Descent is made from the Unison to the Octave or Diapason; and by the same Progression to as many Octaves as there may be Occasion. These are different, according to the different Kinds of Music, viz. Enharmonic, Chromatic, and Diatonic, and the several Colours of the two latter: (all which I shall more conveniently explain by and by); but of these now mention'd, the Diatonic is the most proper and natural Way: The other two, if for Curiosities sake we consider them only by running the Notes of an Octave up or down in these Scales, seem rather a Force upon Nature; yet herein probably might lie the Excellency of the ancient Greeks: But we now use only the Diatonic Kind, intermixing here and there some of the Chromatic,
motic, (and more rarely some of the Enharmonic:) And our Excellency seems to lie in most artificial Composing, and joining several Parts in Symphony or Confort; which they cannot be supposed to have effected, at least in so many Parts as we ordinarily make, because (as is generally affirm'd of them) they own'd no Conords besides Eighth, Fifth, and Fourth, and the Compounds of these.

F. Kircher (cited also by Gassendus without any Mark of Dissent) is of Opinion, that the ancient Greeks never used Confort Music, i.e. of different Parts at once, but only Solitary, for one single Voice or Instrument; and, that Guido Aretinus first invented and brought in Music of Symphony or Confort, both for the one and the other. They apply'd Instruments to Voice, but how they were managed, he must be wiser than I that can tell.

This Way of theirs seems to be more proper (by the elaborate Curiosity and Nicety of Contrivance of Degrees, and by Measures rather than by harmonious Consonancy, and by long-studied Performance) to make great Impressions upon the Fancy, and operate accordingly, as some Histories relate: Ours more sedately affects the Un-
Of Discords and Degrees.

Understanding and Judgment, from the judicious Contrivance and happy Composition of Melodious Consort. The One quietly, but powerfully, affects the Intellect by true Harmony; the Other, chiefly by the Rythmus, violently attacks and hurries the Imagination. In fine, upon the natural Grounds of Harmony (of which I have hitherto been treating) is founded the Diatonic Music; but not so, or not so regularly, the Chromatic or Enharmonic Kinds. Take this following View of them.

The Ancients ascended from the Unison to an Octave by two Systems of Tetrachords or Fourths. These were either Conjunct, when they began the Second Tetrachord at the Fourth Chord, viz. with the last Note of the first Tetrachord, and which being so join'd, constituted but a Seventh; and therefore they assumed a Tone beneath the Unison (which they therefore call'd Proslambanomenos) to make a full Eighth.

Or else the two Tetrachords were disjunct, the Second taking its beginning at the Fifth Chord, there being always a Tone Major between the Fourth and Fifth Chords. So the Degrees were immediately apply'd.
to the Fourths, and by them to the Octave; and were different according to the different Kinds of Music. In the common Diatonic Genus the Degrees were Tone and Semitone; Intervals more Equal and Easy, and Natural. In the common Chromatic, where the Degrees were Hemitones and Trihemitones, the Difference of some of the Intervals was greater: But the greatest Difference, and consequently difficulty, was in the Enharmonic Kind, using only Diësis, or quarter of a Tone, and Ditone, as the Degrees whereby they made the Tetra-chord.

And to constitute these Degrees, some of them, viz. the Followers of Aristoxenus, divided a Tone Major into Twelve equal Parts, i.e. supposed it so divided; Six of which being the Hemitone, (viz. half of it) made a Degree of Chromatic Toniaum; and Three of them, or a quarter, call’d Diësis, a Degree Enharmonic. The Chromatic Fourth rose thus, viz. from the first Chord to the second was a Hemitone; from the second to the third, a Hemitone; from the Third to the Fourth, a Trihemitone; or as much as would make up a just Fourth. And this last Space (in this case) was accounted as well as either of the other, but one Degree or undivided Interval. And they
they call'd them Spiss Intervals [πυξί] when two of those other Degrees put together, made not so great an Interval as one of these; as, in the Enharmonic Tetra-chord, two Dieses were less than the remaining Ditone, and in the common Chromatic, two Hemitone Degrees were less than the remaining Tribemitone Degree.

Then for the Enharmonic Fourth, the first Degree was a Diesis, or quarter of a Tone; the second also Three of those Twelve Parts, viz. a Diesis; the third a Ditone, such as made up a just Fourth. And this Ditone (tho' so large a Degree) being consider'd as thus placed (in the Enharmonic Tetrachord) was likewise to them but as one uncompounded or entire Interval.

These were the Degrees Chromatic and Enharmonic: Tho' they also might be placed otherwise, i.e. the greater Degree in these may change its place, as the Hemitone (the less Degree) doth in the Diatonic Genus; and from this Change chiefly arose the several Moods, Dorian, Lydian, &c. From all which, their Music no doubt (tho' it be hard to us to conceive) must afford extraordinary Delight and Pleasure, if it did bear but a reasonable Proportion
to their infinite Curiosity and Labour. And as we may suppose it to have differ’d very much from that which now is, and for several Ages hath been used; so consequently we may look upon it as in a manner lost to us.

In prosecution of my Design, I am only, or chiefly, to insist on the other Kind of Degrees, which are most proper to the Natural Explanation of Harmony, viz. the Degrees Diatonic; which are so call’d, not because they are all Tones; but because most of ’em, as many as can be, are such; viz. in every Diapason five Tones and two Hemitones. Upon these, I say, I am to insist, as being, of those before mention’d, the most Natural and Rational.

Digression.

But before we proceed, it may perhaps be a Satisfaction to the Reader, after what has been said, to have a little better Prospect of the ancient Greek Music, by some general Account; not of their whole Doctrine, but of that which relates to our present Subject, viz. their Degrees, and Scales of Harmony, and Notes.
Of Discords and Degrees.

First then, take out of Euclid the Degrees according to the three Genera; which were, Enharmonic, Chromatic, and Diatonic; which Kinds have six Colours (as they call’d them). Euclid, Introd. Harm. pag. 10.

The Enharmonic Kind had but one Colour, which made up its Tetrachord by these Intervals; a Diesis (or quarter of a Tone) then such another Diesis, and also a Ditone incomposit.

The Chromatic had three Colours, by which it was divided into Molle, Sescuplum, and Toniaum.

1st. Molle, in which the Tetrachord rose by a Triental Diesis (four of those twelve Parts mention’d before) or third part of a Tone; and another such Diesis; and an incomposit Interval, containing a Tone and half, and third part of a Tone: And it was call’d Molle, because it hath the leaf, and consequently most enervated Spiss Intervals within the Chromatic Genus.

2d. Sescuplum, by a Diesis which is Sesquialtera to the Enharmonic Diesis, and another such Diesis, and an Incomposit Interval of seven Dieses Quadrantal, viz. each being three Duodecimals of a Tone.

H 4

3d. To-
3d. *Toniaeum*, by a Hemitone, and Hemitone and Tribemitone; and is call'd *Toniaeum* because the two *Spiss* Intervals make a *Tone*. And this is the ordinary *Chromatic*.

**The Diatonick** had two Colours; it was *Molle* and *Syntonum*.

1st. *Molle*; by a Hemitone, and an *Incomposit* Interval of three Quadrantal *Dieses* and an Interval of five such *Dieses*.

2d. *Syntonum*, by a Hemitone and a *Tone*, and a *Tone*. And this is the common *Diatonic*.

To understand this better, I must re-assume somewhat which I mention'd, but not fully enough before. A *Tone* is suppos'd to be divided into twelve least parts, and therefore a *Hemitone* contains six of those Duodecimal (or twelfth) parts of a *Tone*; a *Diesis Trientalis* 4, *Diesis Quadrantalis* 3, the whole *Diatessaron* 30. And the *Diatessaron* in each of the three Kinds was made and perform'd upon four *Chords*, having three mean *Intervals* of *Degrees*, according to the following *Numbers* and *Proportions* of those thirty Duodecimal parts.

*Enhar-*
Of Discords and Degrees.

Enharmonic, \[\text{by 3, and 3, and 24}\]
\[\text{by 4, and 4, and 22}\]

Chromatic, \[\text{by } 4\frac{1}{2}, \text{ and } 4\frac{1}{2}, \text{ and } 21\]

Hemiolion, \[\text{by 6, and 6, and 18}\]

Sescuplum, \[\text{by 6, and 9, and 15}\]

Molle, \[\text{by 6, and 12, and 12}\]

Tonicum, \[\text{by 6, and 12, and 12}\]

Enharmonic, \[\text{by 3, and 3, and 24}\]

Chromatic, \[\text{by 4, and 4, and 22}\]

Hemiolion, \[\text{by } 4\frac{1}{2}, \text{ and } 4\frac{1}{2}, \text{ and } 21\]

Sescuplum, \[\text{by 6, and 6, and 18}\]

Molle, \[\text{by 6, and 9, and 15}\]

Tonicum, \[\text{by 6, and 12, and 12}\]

To each of these Kinds, and the Moods of them, they fitted a perfect System or Scale of Degrees to Disdiapason; as in the following Example taken out of Nichomachus; to which I have prefixed our modern Letters.

E. Nichomacho, pag. 22.

<table>
<thead>
<tr>
<th>A</th>
<th>Nete Hyperbolaon.</th>
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<tr>
<td>G</td>
<td>Paranete Hyperbolaon.</td>
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<td></td>
<td>{Enharm. Chro. Diat.}</td>
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<tr>
<td>E</td>
<td>Nete Diezeugmenon.</td>
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<tr>
<td>D</td>
<td>Paranete Diezeugmenon.</td>
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<tr>
<td>C</td>
<td>Trite Diezeugmenon.</td>
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<tr>
<td>B</td>
<td>Paramese.</td>
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D | Nete
In this Scale of Disdiapason you see the Mese is an Octave below the Nete Hyperbolaxon, and an Octave above the Proslambanomenos: And the Lichanos, Parypate, Parenete, and Trite, are changeable; as upon our Instruments are the Seconds, and Thirds, and Sixths, and Sevenths: The Proslambanomenos, Hypate, Mese, Paramese, and Nete, are immutable; as are the Unison, Fourths, Fifths, and Octaves.

Now from the several Changes of these Mutable Chords chiefly arise the several Moods (some call'd them Toncs) of Music, of which Euclid sets down Thirteen; to which were joyn'd two more, viz. Hyperaolian
Of Discords and Degrees.

peræolian and Hyperlydian; and afterwards Six more were added.

I shall give you, for a Taste, Euclid's Thirteen Moods.

Euclid. p. 19.

Hypermixolydians, five Hyperphrygians.
Mixolydians acutior, five Hyperiaustrians.
Mixolydians gravior, five Hyperdorians.
Lydians acutior.
Lydias gravior, five Æolians.
Phrygians acutior.
Phrygians gravior, five Iaslians.
Dorians.
Hypolydians acutior.
Hypolydians gravior, five Hypoaolians.
Hypophrygians acutior.
Hypophrygians gravior, five Hypoiaslians.
Hypodorians.

Of these the most grave, or lowest, was the Hyperdorian Mood, the Prosambanomenos whereof was fix’d upon the lowest clear and firm Note, of the Voice or Instrument that was suppos’d to be of the deepest settled Pitch in Nature, and adapted freely to express it: And then all-along from Grave to Acute the Moods took their Ascent by Hemitones, each Mood being a Hemi-
Of Discords and Degrees.

Hemitone higher or more acute than the next under it. So that the Proslambanomenos of the Hypermixolydian Mood was just an Eighth higher than that of the Hypodorian, and the rest accordingly.

Now each particular Chord in the preceding Scale had two Signs or Notes [ομοιεία] by which it was characteriz’d or describ’d in every one of these Moods respectively; and also for all the Moods in the several Kinds of Music; Enharmonic, Chromatic, and Diatonic; of which two Notes, the upper was for reading [λέξις] the lower for percussion [χρώσ] one for the Voice, the other for the Hand. Consider then how many Notes they used; 18 Chords severally for 13 Moods (or rather 15, taking in the Hyperaolian and Hyperlydian, which are all describ’d by Alypius) and these suited to the three Kinds of Music. So many Notes, and so appropriated, had the Scholar then to learn and conn who studied Music. Of these I will give you in part a View out of Alypius.

Notes
Notes of the Lydian Mood in the Diatonic Genus.

7.7.R.Φ.C.P.M.I.Θ
E.Γ.L.F.C.U.N.<V.

Γ.U.Z.E.U.Θ.L.M.I.
N.Z.C.U.Z.Λ.Π.Ν.<

1. Proslambanomenos. \{ Zeta imperfect, and Tau jacent.  
2. Hypate Hypaton. \{ Gamma averted, and Gamma right.  
3. Parhypate Hypaton. \{ Beta imperfect, and Gamma inverted.  
5. Hypate Meson. — Sigma, and Sigma.  
8. Diese. — Iota, and Lamda jacent  

9. Trite
108  Of Discords and Degrees.

9  Trite Synemmenon. \{\Theta\eta\alpha, and \Lambda\mu\beta\deta\ inverted.
10  Synemmenon Diatonos.  \Gamma\mbox{mma, and }N\upsilon.
11  Nete Synemmenon. \{\Omega squared, lying supine upwards, and \Zeta.
12  Paramese.  \{\Zeta and \Pi jacent.
13  Trite Diezeugmenon \{E squared, and \Pi inverted.
14  Diezeugmenon Diatonos. \{\Phi jacent, and a careless \E\eta\ (\upsilon) drawn out.
15  Nete Diezeugmenon. \{\Upsilon looking down, \& \Alpha, left half, looking upward.
16  Trite Hyperbolaeon. \{\M\eta, and \Pi lengthened, with an Acute above.
17  Hyperbolaeon Diatonos. \{\Iota, and \Lambda\mu\beta\deta jacent, with an Acute above.

The Numeral Figures I have added under the Signs (or Marks) only for Reference to the Names of the Notes signified by them, to have describing them twice.
Of Discords and Degrees.

Notes of the Æolian Mood in the Diatonic Genus.

\[ \text{H.Y.T.X.T.C.O.K.I.} \]
\[ \text{E.H.G.Y.T.C.K.A.} \]
\[ 1 2 3 4 5 6 7 8 9 \]

\[ \text{Z.A.H.Z.A.X.Θ.O.K.} \]
\[ \text{L.A.Δ.C.Λ.Π.Κ.Λ.} \]
\[ 10 11 12 13 14 15 16 17 18 \]

1 Proslambanomenos. \( \{ \) Eta (H) imperfect averted, and Equadrate averted.

2 Hypate Hypaton, &c: \( \{ \) Delta inverted, and Tau jacent, averted, &c.

Aristides (Pag. 91.) enumerates and describes all the Variations of every Letter in the Greek Alphabet; by which the Signs or Notes above mention'd, and those of the other Moods, were contriv'd out of them.
Of Discords and Degrees.

They are in all 91; including the Proper Letters: I shall not describe, but only number them.

Out of

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I shall only add a Word or two concerning their ancient Use of the Words *Diastem* and *System*. *Diastem* signifies an Interval or Space; *System*, a Conjunction or Composition of Intervals. So that, generally speaking, an *Octave*, or any other *System*, might be truly call’d a *Diastem*, and very frequently used to be so call’d, where there was no occasion of Distinction. Tho’ a *Tone*, or *Hemitone*, could not be call’d a *System*; for when they spoke strictly, by a *Diastem* they understood only an Incom-
Incompositt Degree, whether Diesis, Hemitone, Tone, Sesquitone, or Ditone; for the two last were sometimes but Degrees, one Enharmonic, the other Chromatic. By System they meant, a Comprehensiffive Interval, compounded of Degrees, or of less Systems, or of both. Thus a Tone was a Diastem, and Diatessaron was a System, compounded of Degrees, or of a 3d and a Degree. Diapason was a System, compounded of the lesser Systems, 4th, and 5th; or 3d and 6th; or of a Scale of Degrees: And the Scale of Notes which they used, was their Greateft, or Perfect System. Thus with them, a 3d Major, and a 3d Minor, in the Diatonic Genus, were (properly speaking) Systems; the former being compounded of two Tones, and the latter of three Hemitones, or a Tone and Hemitone: But in the Enharmonic Kind, a Ditone was not a System, but an Incompositt Degree; which added to two Dieses, made up the Diatessaron: And in the Chromatic Kind, a Tribemitone was the like; being only an Incompositt Diastem, and not a System.

But to return from this Digression (which is not so much to my Purpose, as to gratify the Reader's Curiosity) and continue our Discourse according to Nature's Guidance, upon the Diatonic Degrees. It was
was said, that there are five *Tones* and two *Hemitones* in every *Diapason*. Now the reason why there must be two *Hemitones*, is, because an 8th is naturally composed of, and divided into 5th and 4th; and a Fifth is three *Tones* and a half; a Fourth two *Tones* and a half; and the Ascent, by Degrees, must pass by Fourth and Fifth; which are always unchangeable, and keep the same distance from *Unison*; and a just *Tone Major* of 9 to 8 always between them. Therefore the *Diapason* has not an Ascent of six *Tones*, but of five *Tones* and two *Hemitones*, one *Hemitone* being placed in each Fourth Disjunct; in either of which Fourths, the Degrees may be alter'd by placing the *Hemitone* in the First, or Second, or Third Degree of either. As, *MI, FA, Sol, La, La, MI, FA, Sol, Sol, La, MI, FA*. If this be done in the former *Tetrachord*, then is chang'd the Second, or Third *Chord*; if in the other Disjunct *Tetrachord*, then the Sixth, or Seventh is chang'd: The Fourth and Fifth being stable and immutable, by them we naturally divide the *Diapason*: The Second, Third, Sixth, and Seventh are alterable, as *Minor*, and *Major*, according to the Place of the *Hemitone*.

*These Tones* and *Hemitones* thus placed, are the Degrees or Notes by which
Of Discords and Degrees.

an Ascent or Descent is made from the Unison to the Octave, or thro' any other System, giving all the Concords their just Measures or Rations; and without which, we could neither Measure, nor Divide, nor well Practise, to learn the greater Intervals or Systems.

As we Naturally by the Judgment of our Ear, own, and rest in the Octave, as the chief Consonant; so we do as Naturally (without Study or Skill in Music) measure the System of a Diapason by these Diatonic Degrees; and can do no otherwise. We cannot with our Voice, without infinite Study, frame to run up or down eight Notes, without such a Mixture of Tones and Hemitones; and we do it easiest when we avoid Tritones. We see it in a Ring of Bells, of which the compleatest and most pleasant is a Peal of Six; which are best sorted to have the Hemitone in the midst; i.e. between the Third and Fourth, both in Ascending and Descending; and then there will be no Tritone: Ex. gr. La, Sol, Fa, Mi, Re, Ut. Where all Ascents and Descents are made by just Diatessarons. Ut, Re, Mi, Fa, Re, Mi, Fa, Sol. Mi, Fa, Sol, La. Or downwards; La, Sol, Fa, Mi. Sol, Fa, Mi, Re. Fa, Mi, Re, Ut.

I 3

AND
AND this is so Natural that it pleaseth all Ears; and if they should be disposed in any other Order, it would be so disagreeable, that any Rustick or unlearn’d Ear, of such as know not what a Tritone is, would be able to judge, and find a Dislike of it. But then, how much more, if the Ring of Bells were dispos’d by Chromatic or Enharmonic Degrees, constituting the Diatessarons? how absurd and uncouth it would appear! The practice of those kinds therefore, and in such a manner, seems to be (as has been said) a Violence upon Nature, and only for Curiosity.

IN Diatonic Music there is but one sort of Hemitone amongst the Degrees, call’d Hemitone Major, whose Ration is 16 to 15; being the Difference, and making a Degree between a Tone Major and Third Minor; or between a Third Major and a Fourth.

There are two sorts of Tones; viz. Major, and Minor. Tone Major (9 to 8) being the Difference between a Fourth and Fifth: And Tone Minor (10 to 9) which is the Difference between Third Minor and Fourth. But both the Tones arising (as hath been said) out of the Partition of a Third Major, in like manner as 5th and 4th do by the Partition of an 8th: I may (with
submifion) make the following Remark; wherein, if I be too bold, or be mistaken, I shall beg the Reader's Pardon.

The ancient Greek Masters found out the Tone by the Difference of a Fourth and Fifth, subtracting one from the other: But had they found it also (and that more Naturally) by the Division of a Fifth; first into a Ditone and Sesquitone, and then by the like proper Division of a true Ditone (or Third Major) into its proper Parts; they must have found both Tone Major and Tone Minor. Euclid rests satisfied, that Inter super-particulare non cadit Medium. A super-particular Ration cannot have a Mediety; viz. in whole Number: Which is true in its Radical Numbers. But had he doubled the Radical Terms of a Super-particular, he might have found Mediums most Naturally and Uniformly dividing the Systems of Harmony; ex. gr. The Duple Ration 2 to 1, as the Excess is but by an Unity, has the Nature of Super-particular: but 2 to 1, the Terms being dupled, is 4 to 2; where 3 is a Medium, which divides it into 4 to 3 (4th) and 3 to 2 (5th). Again, 3 to 2, dupling each Term, is 6 to 4; and in the same manner gives the two Thirds, viz. 6 to 5, (3d Minor) and 5 to 4, (3d Major). Likewise the 3d Major, 5 to 4, I 3 dupled
dupled as before, 10 to 8, gives the two Tones; i. e. 10 to 9, Tone Minor, and 9 to 8, Tone Major.

And it seems to be a Reason why the Ancients did not discover and use the Tone Minor, and consequently not own the Di-tone for a Concord; because they did not pursue this Way of dividing the Systems. Altho' Euclid had a fair Hint to search further, when he measured the Diapason by six Tones [Major] and found them to exceed the Interval of Diapason.

The Pythagoreans, not using Tone Minor, but two equal Tones Major, in a Fourth, were forced to take a lesser Interval for the Hemitone; which is call'd their Limma, or Pythagorean Hemitone; and, which added to those two Tones, makes up the Fourth: 'Tis a Comma less than Hemitone Major, (16 to 15) and the Ration of it is 256 to 243.

Yet we find the later Greek Masters, Ptolemy, to take Notice of Tone Minor; and Aristides Quintilianus, to divide a Sesquioctave Tone (9 to 8) by dupling the Terms of the Ration thereof into two Hemitones; 18 to 17, and 17 to 16. And those again, by the same Way, each into two Dieses;
Of Discords and Degrees.

Dieses; 36 to 35, 35 to 34; the Division of 18 to 17, the less Hemitone: And 34 to 33, and 33 to 32; the Parts of 17 to 16, the greater Hemitone. But yet, none of these were the Complement of two Sesquioctave Tones to Diатessarion: but another Hemitone, whose Ratio is about 20 to 19; not exactly, but so near it, that the Difference is only 1216 to 1215; both which together make the Limma Pythagoricum.

But I no where find, that they thus divided the Fifth and Third major, but rather seem'd to dislike this Way, because of the Inequality of the Hemitones and Dieses thus found out; and chose rather to constitute their Degrees by the Sesquioctave Tone, and those Duodecimal suppos'd-equal Divisions of it. But to return.

There are, you see, three Degrees Diatonic; viz. Hemitone major, Tone minor, and Tone major. The first of these some call Degree minor; the second, Degree major; the third, Degree maxim. Now these three sorts of Degrees are properly to be intermix'd, and order'd, in every Ascent to an Eighth, in relation to the Key, or Unison given, and to the Affections of that Key, as to Flat and Sharp, in our Scale of Music; so, that the Concords may be all true,
true, and stand in their own settled Ration. Wherefore if you change the Key, they must be changed too; which is the reason why a Harpsichord, whose Degrees are fixed; or a fretted Instrument, the Frets remaining fix'd, cannot at once be set in Tune for all Keys: For, if you change the Key, you withal change the Place of Tone minor, and Tone major, and fall into other Hemitones that are not proper Diatonic Degrees, and consequently into false Intervals.

You may fully see this, if you draw Scales of Ascent fitted to several Keys (as are here inserted) and compare them. For an Example of this, Take the first Scale of Ascent to Diapason [I] viz. upon C Key Proper, by Diatonic Degrees; (making the first to be Tone minor, as convenient for this Instance) intermixing the Chromatic and other Hemitones, as they are usually placed in the Keys of an Organ; i.e. run up an Eighth upon an Organ (tuned as well as you can) by Half-Notes, beginning at C Sol fa ut, and you will find these Measures. The Proper Degrees standing right, as they ought to be, being describ'd by Breves; the other by Semibreves: The Breves representing the Tones of the broad Gradual Keys of an Organ; the Semibreves repre-
representing the narrow Upper Keys, which are usually call'd Musics. And let this be the first Scale, and a Standard to the rest.

Then draw a second Scale [II] running up an Eighth in like manner; but let the Key, or First Note be D Sol re, with a Flat Sixth, on the same Organ standing tuned as before; which Key is set a Note (or Tone Minor) higher than the former.

Draw also a third Scale [III] for D Sol re Key with Sharps, viz. Third and Seventh Major; i.e. F, and C, sharp.

In the First of these Scales, the Degrees (express'd by Breves) are set in good and natural Order.

In the Second Scale (changing the Key from C to D) you will find the Second, Fourth, and Sixth, a Comma (81 to 80) too much; but between the Fourth and Fifth, a Tone Minor, which should be always a Tone Major. So, from the Fourth to the Eighth, is a Comma short of Diapente, and from the Sixth, a Comma short of Third Major. And this, because in this Scale the Degrees are misplaced.
The Third Scale makes the Second, Fourth, and Sixth, from the Unison, each a Comma too much; and from the Octave, as much too little. In it, the third Degree, between $\#F$ and $G$, is not the Proper Hemitone, but the Greatest Hemitone, 27 to 25. And all this, because in this Scale also the Degrees are misplaced; and there happen (as you may see) three Tones Minor, and but two Major; the deficient Comma being added to the Hemitone.

I have added one Example more, of a Fourth Scale; [ IV ] viz. beginning at the Key $\#C$; with the like Order of Degrees as in the first Scale (from the Note C $\#$) upon the same Instrument, as it stands tuned after the first Scale: And this will raise the first Scale half a Note higher.

In this Scale, all the Hemitones are of the same Measure with those of the first Scale respectively.

And the Intervals should be the same with those of the first Scale; which has Third, Sixth, Seventh, Major.

But in this fourth Scale, the first Degree, from $\#C$ to $bF$, is Tone major, and Diesis;
Dieſis; as being compounded of 16 to 15, and 27 to 25.

The Second Degree from B E to F, is Tone Minor; therefore the Ditone, made by these two Degrees, is too much by a Dieſis, (128 to 125) and as much too little the Tribemitone, from the Ditone to the Fifth.

The Third Degree, from F to †F, is a Minor Hemitone, 25 to 24; which (tho' a wrong Degree) sets the Diatessaron right.

The Fourth Degree; from †F to †G, is Tone Major, and makes a true Fifth.

The Fifth Degree, from †G to b B, is Tone major, and Dieſis; setting the Hexachord (or Sixth) a Dieſis and Comma too much, or too high. It ought to have been Tone minor.

The Sixth, from b B to C, is Tone minor; too little in that place by a Comma.

The Seventh, from C to †C, is Hemitone Minor; too little by a Dieſis. And so, these two last Degrees are deficient by
Of Discords and Degrees.

A Diesis and Comma; which Diesis and Comma being Redundant (as before) in the fifth Degree, are balanced by the deficiency of a Comma in the sixth Degree, and of a Diesis in the seventh: And so the Octave is set right.

These Disagreements may be better view'd, if we set together, and compare the Degrees of this IV Scale, and those of the I: Where we shall find but one of all the seven Degrees, to be the same in both Scales.

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<th>Scale I</th>
<th>Scale IV</th>
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<tr>
<td>Degrees</td>
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<tr>
<td>1st, Tone minor.</td>
<td>Tone maj. and Diesis.</td>
</tr>
<tr>
<td>2nd, Tone major.</td>
<td>Tone minor.</td>
</tr>
<tr>
<td>3rd, Hemit. major.</td>
<td>Hemitone minor.</td>
</tr>
<tr>
<td>4th, Tone major.</td>
<td>Tone major.</td>
</tr>
<tr>
<td>5th, Tone minor.</td>
<td>Tone maj. and Diesis.</td>
</tr>
<tr>
<td>6th, Tone major.</td>
<td>Tone minor.</td>
</tr>
<tr>
<td>7th, Hemit. major.</td>
<td>Hemitone minor.</td>
</tr>
</tbody>
</table>

And thus 'twill succeed in all Instruments, tuned in order by Hemitones, which are fix'd upon Strings; as Harp, &c. or Strings with Keys; as Organ, Harpsichord, &c. or distinguished by Fretts; as Lute, Viol, &c. for which there is no Remedy, but by some alterations of the Tune of the Strings.
Of Discords and Degrees

Strings in the two former; and of the Space of the Fretts in the latter; as your present Key will require, when you change from one Key to another, in performing Musical Compositions.

Tho' the Voice, in Singing, being free, is naturally guided to avoid and correct those before describ'd Anomalies, and to move in the true and proper Intervals: It being much easier with the Voice to hit upon the right, than upon the anomalous or wrong Spaces.

Much more of this Nature may be found, if you make and compare more Scales from other Keys. You will still find, that, by changing the Key, you do withal change and displace the Degrees, and make use of Improper Degrees, and produce Incongruous Intervals.

For, instead of the Proper Hemitone, some of the Degrees will be made of other sort of Hemitones; amongst which chiefly are these two: viz. Hemitone Maxim. 27 to 25; and Hemitone Minor, or Chromatic, 25 to 24. Which Hemitones constitute and divide the two Tones; viz. Tone major, 9 to 8: the Terms whereof tripled, are 27 to 24; and give 27 to 25, and 25 to 24. The
Of Discords and Degrees.

The Tone minor likewise is divided into two Hemitones; viz. Major, 16 to 15; and Minor, 25 to 24.

These two serve to measure the Tones, and are used also when you Divert into the Chromatic Kind. But the Hemitone Degree in the Diatonic Genus, ought always to be Hemitone Major, 16 to 15; as being the Proper Degree and Difference between Tone major and Trihemitone, between Ditone and a Fourth, between Fifth and Sixth minor, and also between Seventh major and Octave.

Music would have seemed much easier, if the Progression of Dividing had reach'd the Hemitones: I mean, if, as by dupling the Terms of Diapason, 4 to 2; it divides in 4 to 3, and 3 to 2; Diatessaron, and Diapente: And the Terms of Diapente dupled, 6 to 4; fall into 6 to 5, and 5 to 4, Third minor, and Third major; and Ditone, or Third major; so dupled, 10 to 8, falls into 10 to 9, and 9 to 8, Tone minor and Tone major: If, I say, in like manner, the dupled Terms of Tone major 18 to 16, thus divided, had given Useful and Proper Hemitones, 18 to 17, and 17 to 16. But there are no such Hemitones found in Harmony, and we are put to seek the Hemitones out of
Of Discords and Degrees.

of the Differences of other Intervals; as we shall have more Occasion to see, when I come to treat of Differences, in Chap. 8.

I may conclude this Chapter, by shewing how all Consonants, and other Concinnous Intervals, are Compounded of these three Degrees; Tone major, Tone minor, and Hemitone major; being severally placed, as the Key shall require.

Tone Major, and \{ joyn'd, \} make \{ 3d Minor. \}
Hemitone Major, \}

Tone Major, and \{ joyn'd, \} make \{ 3d Major. \}
Tone Minor, \}

Tone Major, and \{ joyn'd, \} make \{ 4th. \}
Tone Minor, \}
Hemitone Major, \}

2 Tones Major, \{ joyn'd, \} make \{ 5th. \}
1 Tone Minor, \}
1 Hemitone Maj. \}

2 Tones Major, \{ joyn'd, \} make \{ 6th Minor. \}
1 Tone Minor, \}
2 Hemitones Maj. \}

2 Tones Major, \{ joyn'd, \} make \{ 6th Major. \}
2 Tones Minor, \}
1 Hemitone Maj. \}

3 Tones
BESIDES the Degrees, which, tho’ they constitute and compound all Con-
cords, yet are reckon’d amongst Discords; because every Degree is Discord to each
Chord, to, or from which it is a Degree, either Ascending or Descending, as being a
Second to it: Besides these, I say, there are other Discords, some greater, and some
less.
Of Discords.

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les. The les will be found amongst the Differences in the next Chapter; and are fit, rather to be known as Differences, than to be used as Intervals.

The greater Discords are generally made of such Concors as, by reason of misplaced Degrees happen to have a Comma, or Diesis, or sometimes a Hemitone too much, or too little; and so become Discords, most of them being of little Use, only to know them, for the better measuring and rectifying the Systems: Yet they are found amongst the Scales of our Music.

Sometimes a Tone Major being where a Tone Minor should have been placed, or a Tone Minor instead of a Tone Major; sometime other Hemitones, getting the place of the Diatonic Hemitone Major, and serving for a Degree, create unapt Discording Intervals: amongst which may be found at least two more Seconds, two more Thirds, two more Sixths, and two more Sevenths. In each of which, one is less, and the other greater, than the true legitimate Intervals, or Spaces of those Denominations; as will be more explain’d in the ensuing Discourse.
Of Discords.

But besides these (or rather amongst them, for I here treat of Degrees as Discords) there are two Discords eminently considerable, viz. Tritone, and Semidiapente. The Tritone, (or false Fourth) whose Ration is 45 to 32, consists of three whole Notes; viz. two Tones Major, and one Minor. The Semidiapente (or false Fifth) 64 to 45; is compounded of a Fourth and Hemitone Major.

And these two divide Diapason, 64 to 32, by the Mediety of 45; And they divide it so near to Equality, that in Practice they are hardly to be distinguish'd, and may almost pass for one and the same: but in Nature, they are sufficiently distinguish'd; as may be seen both by their several Rations, and several Compounding Parts.

I think we may reckon 7ths for Degrees, as well as among the greater Discording Intervals; because they are but Seconds from the Octave, and are as truly Degrees Descending, as the Seconds are in Ascent: tho' they be great Intervals in respect of the Unison, and such as may be here regarded.

These Discords, the Tritone, and Semidiapente; as also, the Seconds, and Sevenths, are
are of very great use in Music, and add a wonderful Ornament and Pleasure to it, if they be judiciously managed. Without them, Music would be much less grateful; like as Meat would be to the Palate without Salt or Sawce. But, the further Consideration of this, and to give Directions when, and how to use 'em, is not my Task, but must be left to the Masters of Composition.

**Discords** then, such as are more apt and useful (*Intervalla Concinnia*) are these which follow.

2d Minor; or, Hemitone Major, 16 to 15.
2d Major; Tone Minor, 10 to 9.
2d Greatest; Tone Major 9 to 8.
7th Minor; 5th & 3d Minor, 9 to 5.
7th Major; 5th & 3d Major, 15 to 8.
Tritone; 3d Maj. & Tone Maj. 45 to 32.
Semidiapente; 4th & Hemit. Maj. 64 to 45.

These are the Simple dissonant apt Intervals within *Diapason*; if you go a further Compass, you do but repeat the same Intervals added to *Diapason*, or *Disdiapason*, or *Tris-diapason*, &c. as, Ex. gr.
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Of Discords.

<table>
<thead>
<tr>
<th>9th</th>
<th>10th</th>
<th>11th</th>
<th>12th</th>
<th>15th</th>
<th>19th</th>
<th>22th</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Diapason</em> with a 2d.</td>
<td><em>Diapason</em> with a 3d.</td>
<td><em>Diapason</em> with a 4th, or</td>
<td><em>Diapason</em> cum Diapente</td>
<td><em>Dis-diapason</em></td>
<td><em>Dis-diapason</em> cum Diapente</td>
<td><em>Tris-diapason</em>, &amp;c.</td>
</tr>
</tbody>
</table>

Here, by the way, the Reader may take a little Diversion, in practising to measure the Rations of some of those Intervals in the foregoing Catalogue of Discords, by comparing them with *Diapason*; as those of the *Sevenths*, which I select, because they are the most distant Rations under *Diapason*; viz. *Seventh minor*, 9 to 5; and *Seventh major*, 15 to 8. Now to find what Degree or Interval lies between these and *Diapason*.

First, 9 to 5 is 10 to 5, wanting 10 to 9 (*Tone minor.*) Next, 15 to 8 is 16 to 8, wanting 16 to 15 (*Hemitone major.*) So the Degree between *Seventh minor* and *Diapason*, is *Tone minor*; and between *Seventh major* and *Diapason*, is *Hemitone major.*

Then
Of Discords.

Then he may exercise himself in a Survey of what Intervals are compriz’d in those several Sevenths, and of which they are compounded.

First, 9 to 5 comprizeth 9 to 8, and 8 to 5: Or, 9 to 8, 8 to 6, and 6 to 5. Next, 15 to 8 contain 15 to 12, 12 to 10, 10 to 9, and 9 to 8: Or, 15 to 12, and 12 to 8: Or, 15 to 10, and 10 to 8, &c. I suppose that the Reader, before this, is so perfect in these Rations, that I need not lose Time to name the Intervals express’d by the Mean Rations, contain’d in the foregoing Rations of the Sevenths, which shew of what Intervals the several Sevenths are compounded.

Besides these (by reason of Degrees wrong placed) there are two more Sevenths; [false Sevenths] one, less than the true ones, and another greater. The least compounded of two Fourths, whose Ration is 16 to 9, and wants a Comma of Seventh minor, and a Tone major of Diapason: The other is the greatest, call’d Semidiapason, whose Ration is 48 to 25; being a Diesis more than Seventh major, and wanting Hemitone minor of Diapason.

Now,
Now, first, \(16\) to \(9\) is \(16\) to \(8\) (2 to 1) wanting \(9\) to \(8\); i.e. wanting Tone major of Diapason; and contains \(16\) to \(10\) (8 to 5) and \(10\) to \(9\): Or, \(16\) to \(15\), \(15\) to \(12\) (5 to 4) \(12\) to \(10\) (6 to 5) and \(10\) to \(9\). Next, Semidiapason \(48\) to \(25\), is \(50\) to \(25\), wanting \(50\) to \(48\); i.e. \(25\) to \(24\) (viz. Hemitone minor) of Diapason.

And the like happens, as hath been said, to the other Intervals, which admit of major and minor; viz. Seconds, Thirds, and Sixths. The Fourth, and Fifth, and Eighth ought always to remain immutable; tho' they may suffer too sometimes, and incline to Discord, if we ascend to them by very wrong Degrees; as you may see in the \(II^{d}\) Scale in the foregoing Chapter; where the Fourth having two Tones major, is a Comma too much.

All these Intervals may be subject to more Mutations, by more absurd placing of Degrees, or of Differences of Degrees; but it is not worth the Curiosity to search farther into them: The Reader may take Pleasure, and sufficiently exercise himself, in comparing and measuring these which are already laid before him.
Of Discords.

But to return from this Digression. There are many unapt Discords, which may arise by continual Progression of the same Concords; i.e. by adding (for Example) a Fourth to a Fourth, a Fifth to a Fifth, &c. for 'tis observable, that only Diapason added (as oft as you please) to Diapason, still makes Concord: But any other Concord, added to it self, makes Discord.

You will see the Reason of it, when you have consider'd well the Anatomy (as I may call it) of the Constitutive Parts of Diapason; which contains, and is compos'd of seven Spaces of Degrees, or of Fourth and Fifth, or of Thirds and Sixths, or of Seconds and Sevenths; which must all keep their true Measures and Rations belonging to them, and otherwise are easily and often disorder'd.

Then, consider Diapason as constituted of two Fourths disjunct, and a Tone major between 'em. And this last is most needful to be very well consider'd; as most plainly shewing the Reasons of those Anomalies, or irregular Intervals, which are produced by changing the Key, and consequently giving a new and wrong Place to this odd Tone major, which stands in the middle
midst of Diapason, between the two Fourths disjunct.

Every Fourth must consist of one Tone major, one Tone minor, and one Hemitone major, as its Degrees, placing them in what Order you please; whose Rations, added together, make the Ration of Diatessaron. And of these same Degrees contain’d in the Fourth, are made the two Thirds, which constitute the Fifth. Tone major and Hemitone major make the less Third, or Trihemitone; Tone major and Tone minor make the greater Third, or Ditone; Trihemitone and Ditone make Diapente; Trihemitone and Tone Minor (as likewise Ditone and Hemitone major) make Diatessaron.

Now this Tone Major, that stands in the middle of Diapason, between the two Fourths, which it disjoins; and the Degrees requir’d to the Fourths, will not in a fixed Scale stand right, when you alter your Key, and begin your Scale of Diapason from another Note: For that which was the Fifth, will now be the Fourth, or Sixth, &c. and then the Degrees will be disorder’d, and create some discordant Intervals. If you continue conjunct Fourths, there will be a Defect of Tones Major; if you continue conjunct Fifths, there will be too many Tones Major
Major in the Systems produced. And if a 
Tone Major be found, where it ought to 
have been a Tone Minor; or a Minor instead 
of a Major; that Interval will have a Com-
ma too much, or too little. And so like-
wise will from a wrong Hemitone be found 
the Difference of a Diesis. And these two, 
Comma and Diesis, are so often redundant, 
or deficient, according as the Degrees hap-
pen to be disorder’d or misplaced; that 
thereby the Difficulties of fixing half-Notes 
of an Organ in tune for all Keys, or giving 
the true Tune by Fretts, become so insu-
perable.

You see, that in every Space of an 
Eighth, there are to be three Tones major, 
and two Tones minor, and two Hemitones 
major: One Tone major between the Dio-
tessaron and Diapente, and a Tone major, 
a Tone minor, and Hemitone major in each of 
the disjunct Fourths.

These are the proper Degrees by 
which you should always Ascend or De- 
scend thro’ Diapason, in the Diatonic Kind; 
which Diapason being the compleat System, 
containing all primary Simple Harmonic In-
tervals that are; (and for that reason call’d 
Diapason;) you may multiply it, or add it 
to its self as oft as you please, as far as Voice 
or
or Instrument can reach, and it will still be Concord, and cannot be disorder'd by such Addition; because every of them will contain (however placed) just three *Tones major*, two *Tones minor*, and two *Hemitones major*.

Whereas, if you add any other Interval to itself, the Degrees will not fall right, and it will be Discord, because all Concords are compounded of unequal Parts, as hath been shewn before; and if you carry them in equal Progression, they will mix with other Intervals by incongruous Degrees, and those disorder'd Degrees will create a dissonant Interval. See the following Scheme of it.

\[
\begin{align*}
2 & & 3\text{ds minor} & & \begin{cases}
5\text{th, wanting Hemit. min.} \\
5\text{th, and Hemit. minor.}
\end{cases} \\
2 & & 3\text{ds major} & & \begin{cases}
8\text{th, wanting Tone major.} \\
8\text{th, and Tone major.}
\end{cases} \\
2 & & 4\text{ths} & & \begin{cases}
8\text{th, and Ditone, & Diesis.} \\
8\text{th, and 4th, & Hemit. min.}
\end{cases} \\
2 & & 5\text{ths} & & \begin{cases}
6\text{ths minor} \\
6\text{ths major}
\end{cases}
\end{align*}
\]

To which may be added, That

\[
\begin{align*}
2 & & \text{Tones minor} & & \begin{cases}
\text{Ditone, wanting a Comma.} \\
\text{Ditone, and a Comma.}
\end{cases} \\
2 & & \text{Tones maj.}
\end{align*}
\]

It was said above, That *Diapason* may be added to itself as oft as you please, and there
there will be no Disorder, because every one of 'em will still retain the same Degrees of which the first was compos'd: But it is not so in other Concord; of which I will add one more Example, because of the Use which may be made of it.

Make a Progression of four Diapente's, and, as was shew'd in the Fifth Chapter, it will produce Disdiapason, and two Tones major, which is a 17th, with a Comma too much; because in that Space there ought to be just seven Tones major, and five Tones minor; whereas in four Fifths continued, there will be found eight Tones major, and but four Tones minor. So that a Tone major, getting the Place of a Tone minor, there will be in the whole System a Comma too much. One of these major Tones should have been a Tone minor, to make the Excess above Disdiapason a just Ditone.

On the other side, if you continue the Ration of four Diatessarons, there will be a Tone minor, instead of a Tone major; and consequently a Comma deficient in constituting Diapason and Sixth minor. For since every Fourth must consist of the Degrees of Tone minor, one Tone major, one Hemitone major; it follows, that if you continue four Fourths, there will be four Tones minor, four Tones
Tones major. and four Hemitones major: Whereas in the Interval of Diapason with Sixth minor, there ought to be five Tones major, and but three minor.

By this you may see the Reason, why, to put an Organ or Harpsichord into more general useful Tune, you must tune by Eighths and Fifths; making the Eighths perfect, and the Fifths a little bearing downward; i.e. as much as a quarter of a Comma, which the Ear will bear with in a Fifth, tho’ not in an Eighth. For Example, begin at C fa ut; make C Sol fa ut a perfect Eighth to it, and G Sol re ut a bearing Fifth; then tune a perfect Eighth to G, and a bearing Fifth at D La sol re; and from thence downwards (that you may keep towards the middle of the Instrument) a perfect Eighth at D Sol re: And from thence a bearing Fifth up at A; and from A, a perfect Eighth upwards, and bearing Fifth at E La mi. From E an Eighth downwards; and so go on, as far as you are led by this Method, to tune all the middle part of the Instrument; and at last fill up all above, and below, by Eighths from those which are settled in Tune, according to the Scheme annex’d; observing (as was said) to tune the Eighths perfect, and the Fifths a little bearing flat; except in the three
last Bards of Fifths, where the Fifths begin to be taken downward from C, as they were upwards in all before: Therefore, as before, the Fifth above bore downward; so here, the Fifth below must bear upward, to make a bearing Fifth: but that being not so easie to be judg’d, alter the Note below, till you judge the Note above to be a bearing Fifth to it. This will rectifie both those Anomalies of Fifths and Fourths: For the Fifth to the Unison, is a Fourth to the Octave; and what the Fifth loseth by Abatement, the Fourth will gain: Which doth in a good Degree rectifie the Scale of the Instrument. Taking Care withal, that what Anomalies will still be found in this Hemitonic Scale, may, by the Judgment of your Ear, in tuning, be thrown upon such Chords as are leaft used for the Key; as ♭G, bE, &c. even which the Ear will bear with, as it doth with other Discords in binding Passages; if so, you close not upon them. But the other Discords, so used, are most Elegant; these only more Tolerable.
Of Differences.

C H A P. VIII.

Of Differences.

All Rations and Proportions of Inequality, have a Difference between them, when compared to one another; and consequently the Intervals, expressed by those Rations, differ likewise. A Fifth is different from a Fourth, by a Tone Major; from a Third Minor, by a Third Major; so an Eighth from a Fifth, by a Fourth. Of the Compounding Parts of any Interval, one of them is the Difference between the other Part and the whole Interval.

But I treat now of such Differences as are generally less than a Tone, and create the Difficulties and Anomalies occurring in the two foregoing Chapters. I have the less to say of them apart, because I could not avoid touching upon them all-along. 'Twill only therefore be needful, to set before you an orderly View of them. And, first, taking an Account of the true Harmonic Intervals, with their Differences, and the Degrees by which they arise; 'twill be easier to judge of the false Intervals, and of what Concern they are to Harmony.
Table of true Diatonic Intervals within Diapason, with the Differences between them.

<table>
<thead>
<tr>
<th>Compounded of</th>
<th>Their Rations</th>
<th>Their Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hemitone Major.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tone Minor.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tone Major.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3d Minor</td>
<td>16 to 15</td>
<td>25 to 24</td>
</tr>
<tr>
<td>3d Major</td>
<td>10 to 9</td>
<td>18 to 24</td>
</tr>
<tr>
<td>4th.</td>
<td>9 to 8</td>
<td>16 to 24</td>
</tr>
<tr>
<td>5th.</td>
<td>6 to 5</td>
<td>25 to 24</td>
</tr>
<tr>
<td>6th Minor</td>
<td>5 to 4</td>
<td>10 to 9</td>
</tr>
<tr>
<td>6th Major</td>
<td>4 to 3</td>
<td>9 to 8</td>
</tr>
<tr>
<td>7th Minor</td>
<td>3 to 2</td>
<td>16 to 15</td>
</tr>
<tr>
<td>7th Major</td>
<td>8 to 5</td>
<td>25 to 24</td>
</tr>
<tr>
<td>Diapason</td>
<td>5 to 3</td>
<td>27 to 25</td>
</tr>
<tr>
<td>Tritone;</td>
<td>9 to 5</td>
<td>25 to 24</td>
</tr>
<tr>
<td>Semidiapente;</td>
<td>15 to 8</td>
<td>16 to 24</td>
</tr>
<tr>
<td>4th.</td>
<td>2 to 1</td>
<td>2048 to</td>
</tr>
<tr>
<td></td>
<td>64 to 45</td>
<td>2025</td>
</tr>
</tbody>
</table>

Those which arise from the Differences of Consonant Intervals, are call'd Intervalia Concina, and properly appertain to Harmony: The rest are necessary to be known, for making and understanding the Scales of Musick.
Table of false Diatonic Intervals, caused by Improper Degrees; with their Rations and Differences from the true Intervals.

This Mark + stands for more; — for less.

<table>
<thead>
<tr>
<th></th>
<th>Rations</th>
<th>Differences from true</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tribemitone</td>
<td>Least;</td>
<td>Tone minor, and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hemit. major.</td>
</tr>
<tr>
<td></td>
<td>Greatest;</td>
<td>Tone major, and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hemit. max.</td>
</tr>
<tr>
<td>Ditone</td>
<td>Least;</td>
<td>2 Tones minor.</td>
</tr>
<tr>
<td></td>
<td>Greatest;</td>
<td>2 Tones major.</td>
</tr>
<tr>
<td>Fourth</td>
<td>Least;</td>
<td>2 Tones minor, &amp;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hemit. major.</td>
</tr>
<tr>
<td></td>
<td>Greater;</td>
<td>2 Tones maj. &amp;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hemit. maj.</td>
</tr>
<tr>
<td>Fifth</td>
<td>Least;</td>
<td>Less 4th, and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tone major.</td>
</tr>
<tr>
<td></td>
<td>Greater;</td>
<td>Greater 4th, and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tone maj.</td>
</tr>
<tr>
<td>Sixth</td>
<td>Least;</td>
<td>5th, and Hemit. minor.</td>
</tr>
<tr>
<td></td>
<td>Greatest;</td>
<td>5th, and Tone</td>
</tr>
<tr>
<td></td>
<td></td>
<td>major.</td>
</tr>
<tr>
<td>Seventh</td>
<td>Least;</td>
<td>6th major, and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hemit. major.</td>
</tr>
<tr>
<td></td>
<td>Greatest;</td>
<td>6th minor, and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3rd minor.</td>
</tr>
</tbody>
</table>

Here
Of Differences.

Here in this Account may be seen, how frequently the Comma, and the Diesis, Abounding or Deficient, by reason of Misplaced Degrees, occasion Discord in Harmonic Intervals.

The Comma, by reason of a wrong Tone, i.e. too much, when a Tone Major happens where there ought to be a Tone Minor; or too little, when the Tone Minor is placed instead of the Major. And the Diesis is Redundant, or Deficient, by reason of a wrong Hemitone; when the Major happens instead of the Minor, or the contrary: the Diesis being the Difference between them. And if Hemitonium Maximum get in the Place of Hemitonium Majus, the Excess will be a Comma; if in the Place of Hemitone Minor, the Excess will be Comma and Diesis.

And these Anomalies are not Imaginary, or only Possible, but are Real in an Instrument fix'd in Tune by Hemitones; as, Organ, Harpsichord, &c. And the Reader may find some of 'em amongst those four Scales of Diapason; in the Sixth Chapter; to which also more may be added: Out of the First of which, I have selected some Examples, using the common Marks, as before, viz. + for more; and — for less or wanting.
Next, take account of some Differences which constitute several Hemitones.
Of Differences.

<table>
<thead>
<tr>
<th>Difference between</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tone Major, and Hemit. Minor.</td>
</tr>
<tr>
<td>Tone Major, and Hemit. Major.</td>
</tr>
<tr>
<td>2Tones Major, and 4th</td>
</tr>
<tr>
<td>Tone Maj. and Limna,</td>
</tr>
</tbody>
</table>

To which may be added out of Mersennus,

<table>
<thead>
<tr>
<th>Difference between</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tone Minor, and Hemitone Maxim.</td>
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<td>or, Hemitone Minor, and Comma.</td>
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Next, take a farther View of Differences, most of which arise out of the preceding Differences, by which you will better see how all Intervals are Compounded and Differenced, and more easily judge of their Measures.
Table of more Differences.

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<tr>
<th>Difference between</th>
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These

Somewhat \(2^{1}2^{5}\) more than \(2^{1}2^{5}\) to \(2^{1}2^{7}2\).
Of Differences.

These Differences (with some more) are found between several other Intervals; of which more Tables might be drawn, but I shall not trouble the Reader with them. Having here shewn what they are, he may (if he please) exercise himself to examine These by Numbers, and also find out Them; and to some it may be pleasant and delightful: And, for that Reason, I have the more largely insinuated on this part of my Subject, which concerns the Measures, Habitudes, and Differences of Harmonic Intervals.

I shall add one Table more, of the Parts of which these lesser Intervals are compounded; which will still give more Light to the former; and is, in Effect, the same.
I think there scarce needs an Apology for some of these Appellations, in respect of Grammar. That I call Hemitonium, and Hexachordon, Majus and Minus; sometimes Hemitone, and Hexachord, major, and minor. These two last Words are so well adapted to our Language, that there's no English-man but knows them. Therefore when I make Hemitone an English Word, I take major and minor to be so too, and fittest to be join'd with it, without respect of Gender.
CHAP. IX.

Conclusion.

To conclude all. Bodies by Motion make Sound; Sound, of fitly-constituted Bodies, makes Tune: Tune, by Swiftness of Motion is render'd more acute; by Slowness more grave: in proportion to the Measure of Courses and Recourses, of Tremblings or Vibrations of Sonorous Bodies. Those Proportions are found out by the Quantity and Affections of Sounding Bodies; _ex. gr._ by the Length of Chords. If the Proportion of Length _ceteris paribus_ and consequently of Vibrations of several Chords, be commensurate within the Number 6; then those Intervals of Tune are Consonant, and make Concord, the Motions mixing and uniting as they pass: If incommensurate, they make Discord by the jarring and clashing of the Motions. Consonants are within a limited Number, Discords innumerable. But of them, those only here consider'd, which are _as the Greeks term'd them_ ἐμελέα, _Concinnous_, apt and useful in Harmony: Or which, at least, are necessary to be known, as being the
the Differences and Measures of the other; and helping to discover the Reason of Anomalies, found in the Degrees of Instruments tuned by Hemitones.

All these I have endeavour'd to explain, with the manifest Reasons of Consonancy and Dissonancy (the Properties of a Pendulum giving much Light to it) so as to render them easie to be understood by almost all sorts of Readers; and to that end have enlarg'd, and repeated, where I might (to the more intelligent Reader) have compriz'd it very much shorter. But I hope the Reader will pardon that, which could not well be avoided, in order to a full and clear Explanation of that, which was my Design, viz. the Philosophy of the Natural Grounds of Harmony.

Upon the whole, you see how Rationally, and Naturally, all the Simple Conords, and the two Tones, are found and demonstrated, by Subdivisions of Diapason.

2 to 1, i.e. 4 to 2; into 4 to 3, and 3 to 2.
2 to 1, i.e. 6 to 3; into 6 to 5, and 5 to 3.
2 to 1, i.e. 8 to 4; into 8 to 5, and 5 to 4.
2 to 1, i.e. 10 to 5; into 10 to 9, 9 to 8, and 8 to 5.
IN which are the Rations (in Radical, or Least Numbers) of the Octave, Fifth, Fourth, Third Major, Third Minor, Sixth Major, Sixth Minor, and Tone Major, and Tone Minor.

And then, all the Hemitones, and Diesis, and Comma, are found by the Differences of these, and of one another; as hath been shewn at large.

Now, certainly, this is much to be preferred before any Irrational Contrivance of expressing the several Intervals. The Aristoxenian Way of dividing a Tone [Major] into twelve Parts, of which 3 made a Diesis, 6 made Hemitone, 30 made Diatessaron, (as hath been said) might be useful, as being easier for Apprehension of the Intervals belonging to the three Kinds of Musick; and might serve for a least common Measure of all Intervals (like Mr. Mercator's artificial Comma) 72 of them being contain'd in Diapason.

But this Way, and some other Methods of dividing Intervals equally, by Surd Numbers and Fractions, attempted by some modern Authors; could never constitute true Intervals upon the Strings of an Instrument, nor
nor afford any Reason for the Causes of Harmony, as is done by the Rational Way, explaining Consonancy by united Motions, and Coincidence of Vibrations. And tho’ they suppos’d such Divisions of Intervals; yet we may well believe, that they could not make them, nor apply ’em in tuning a Musical Instrument; and if they could, the Intervals would not be true, nor exact. But yet, the Voice offering at those, might more easily fall into the true Natural Intervals. *Ex. gr.* The Voice could hardly express the ancient Ditone of two Tones Major; but, aiming at it, would readily fall into the Rational Consonant Ditone of 5 to 4, consisting of Tone Major and Tone Minor. It may well be rejected as unreasonable, to measure Intervals by Irrational Numbers, when we can so easily discover and assign their true Rations in Numbers, that are minute enough, and easy to be understood.

I did not intend to meddle with the Artificial Part of Musick: The Art of Composing, and the Metric and Rhythmical Parts, which give the infinite Variety of Air and Humour, and indeed the very Life to Harmony; and which can make Musick, without Intervals of Acuteness and Gravity, even upon a Drum; and by which chiefly
chiefly the wonderful Effects of Musick are perform'd, and the Kinds of Air distinguish'd; as, Almand, Corant, Jigg, &c. which variously attack the Fancy of the Hearers; some with Sprightfulness, some with Sadness, and others a middle Way: Which is also improv'd by the Differences of those we call Flat, or Sharp Keys; the Sharp, which take the Greater Intervals within Diapason, as Thirds, Sixths, and Sevenths Major; are more brisk and airy; and being assisted with Choice of Measures last spoken of, do dilate the Spirits, and rouze 'em up to Gallantry and Magnanimity. The Flat, consisting of all the less Intervals, contract and damp the Spirits, and produce Sadness and Melancholy. Lastly, a mixture of these, with a suitable Rhythmus, gently fix the Spirits, and compose them in a middle Way: Wherefore the First of these is call'd by the Greeks Dialectic, Dilating; the Second, Systaltic, Contracting; the Last, Hesychastic, Appeasing.

I have done what I design'd, search'd into the Natural Reasons and Grounds, the Materials of Harmony; not pretending to teach the Art and Skill of Musick, but to discover to the Reader the Foundations of it, and the Reasons of the
Conclusion.

Anomalous Phenomena, which occur in the Scales of Degrees and Intervals; which tho' it be enough to my Purpose, yet is but a small (tho' indeed the most certain, and, consequently, most delightful) Part of the Philosophy of Musick; in which there remain infinite curious Disquisitions, that may be made about it; as, what it is that makes Humane Voices, even of the same Pitch, so much to differ one from another? (For tho' the Differences of Humane Countenances are visible, yet we cannot see the Differences of Instruments of Voice, nor consequently of the Motions and Collisions of Air, by which the Sound is made.) What it is that constitutes the different Sounds of the Sorts of Musical Instruments, and even single Instruments? How the Trumpet, only by the Impulse of Breath, falls into such Variety of Notes, and in the Lower Scale makes such Natural Leaps into Consonant Intervals of Third, Fourth, Fifth, and Eighth. But this, I find, is very ingeniously explicated by an honourable Member of the R. S. and publish'd in the Philosophical Transactions, No 195. Also how the Tube-Marine, or Sea- Trumpet (a Monochord) so fully expresseth the Trumpet; and is also made to render other Varieties of Sounds; as, of a Violin,
lin, and Flageolet, whereof I have been an Ear-witness? How the Sounds of Harmony are receiv'd by the Ear; and why some Persons do not love Musick? &c.

As to this last; the incomparable Dr. Willis mentions a certain Nerve in the Brain, which some Persons have, and some have not. But further, it may be consider'd, that all Nerves are composed of small Fibres; Of such in the Guts of Sheep, Cats, &c. are made Lute-Strings: And of such are all the Nerves, and amongst them, those of the Ear, composed. And, as such, the latter are affected with the regular Tremblings of Harmonic Sounds. If a false String (such as I have before describ'd) transmit its Sound to the best Ear, it displeaseth. Now, if there be found Falseness in those Fibres, of which Strings are made, why not the like in those of the Auditory Nerve in some Persons? And then 'tis no Wonder if such an Ear be not pleas'd with Musick, whose Nerves are not fitted to correspond with it, in commensurate Impressions and Motions. I gave an Instance, in Chap. III, how a Bell-Glass will tremble and echo to its own Tune, if you hit upon it: And I may add, That if the Glass should be irregu-
irregularly framed, and give an uncertain Tune, it would not answer your Trial. In fine, Bodies must be regularly framed to make Harmonic Sounds, and the Ear regularly constituted to receive them. But this by the by; and only for a Hint of Enquiry.

I was saying, That there remain infinite Curiosities relating to the Nature of Harmony, which may give the most Acute Philosopher Business, more than enough, to find out; and which, perhaps, will not appear so easie to demonstrate and explain, as are the Natural Grounds of Consonancy and Dissonancy.

After all therefore, and above all, by what is already discover'd, and by what yet remains to be found out; we cannot but see sufficient Cause to rouse up our best Thoughts, to Admire and Adore the Infinite Wisdom and Goodness of Almighty God. His Wisdom, in ordering the Nature of Harmony in so wonderful a manner, that it surpasseth our Understanding to make a through Search into it, tho' (as I said) we find so much by Searching, as does recompense our Pains with Pleasure and Admiration.

AND
And his Goodness, in giving Musick for the Refreshings and Rejoycings of Man-kind; so that it ought, even as it relates to Common Use, to be an Instrument of our Great Creator's Praise, as He is the Founder and Donor of it.

But much more, as 'tis advance'd and ordain'd to relate immediately to his Holy Worship, when we Sing to the Honour and Praise of God. It is so Essential a Part of our Homage to the Divine Majesty, that there was never any Religion in the World, Pagan, Jewish, Christian, or Mehemetan, that did not mix some Kind of Musick with their Devotions; and with Divine Hymns, and Instruments of Musick, set forth the Honour of God, and celebrate his Praise. Not only, Te decet Hymnus Deus in Sion, (Psal. 65.) but also — Sing unto the Lord all the whole Earth, (Psal. 96.)

And it is that which is incessantly perform'd in Heaven, before the Throne of God, by a General Confort of all the Holy Angels and the Blessed.

In short, we are in Duty and Gratitude bound to bless God, for our Delightful Refresh-
Refreshments by the Use of Musick; but especially, in our Publick Devotions, we are oblig'd by our Religion, with Sacred Hymns and Anthems, to magnifie his Holy Name; that we may at laft find Admission above, to bear a Part in that Blessed Confort, and eternally Sing *Hallelujahs* and *Trisagions* in Heaven.

Δόξα τοῦ Θεοῦ.

FINIS.
RULES for Playing
A
THOROW-BASS,
By the late Famous
Mr. Godfry Keller.

MUSICK consists in Concord; and Discords; the Concord; are Four, and of two kinds, viz. Perfect and Imperfect; the Perfect are the 5th and 8th; the Imperfect the 3d and 6th. The Discords are Three, viz. the 2d, 4th, and 7th; the 9th being the same with the 2d, but differently accompany'd.

The Flat Imperfect 5th is used, either as a Concord or Discord, but most commonly like the latter.

The following Scheme, (the Treble ascending by Semitones) shews all Concord; and Discords, as they stand with regard to the Bass.

By Chords is meant either Concord; or Discords; by Semitones is meant half Notes.

There are other Chords us'd sometimes, as the flat 5th, &c. but those shall be treated of hereafter.

In common Chords which are the 3d, 5th, and 8th avoid the taking two 5ths or two 8ths, together, not being allow'd either in Playing or Composition; and the best way to do it in playing, is to move your Hands contrary one to the other.

When the first common Chord you take is the 3d, the next must be the 5th and 8th, and so vice versa, as the following Scheme will illustrate.

Exam.
Example of Common Chords differently taken.

The Sixth may be taken with the third and eighth, in full Playing the following several ways.

Example of Common Chords and Sixes, taken the several ways above mentioned.

On any Note where nothing is mark'd, common Chords are play'd.

In Sixes must be observ'd that when the Bafs is low, and requires a natural flat 6th, you must play two sixes and one third; if the Bafs is high and requires a natural flat 6th, play two thirds and one sixth; if the 3d or 6th happens to be sharp instead of two sixes or two thirds, play the same.

Play \[ \left\{ \begin{array}{c|c|c} 8 & 3 & 6 \\ 6 & 3 & 8 \end{array} \right\} \]

Also in Divisions where a sixth is required, instead of two thirds, or two sixes, play the same.

A flat or sharp mark'd over or under any Note in the Bafs, signifies a flat or sharp third to be play'd. A flat or sharp mark'd before a Note signifies that Note or Figure to be play'd flat or sharp.
Rules for a Thorow-Bafs.

All Keys are known to be flat or sharp, not by the flats or sharps plac'd at the beginning of a Lesson; but by the third above the Key, for if your third is flat, the Key is flat; if your third is sharp, the Key is sharp.

All sharp Notes naturally require flat Thirds, all flat Notes require sharp Thirds; the same Rule hold as to Sixes.

B, E, and A are naturally sharp Notes in an open Key; F, C, and G are naturally flat Notes in an open Key.

Discords are prepared by Consonants, and resolved into Consonants, which are brought in when a part lies still, and are sometimes used in contrary motion.

There are three sorts of Cadences, or full Closes, as when the Bafs falls a fourth, or rises a fourth, viz. the Common Cadence; the 6th and 4th Cadence; and the great (or fullest) Cadence. Each of these may be accompany'd different ways; as will be seen by the following Examples.

The Common Cadence.

\[
\begin{align*}
\text{The 6th and 4th Cadence.} \quad &\{ 8 \ b7 \ | \ 4 \ \#3 \ | \ 5 \ \#5 \ | \\
& 6 \ 5 \ | \ 8 \ b7 \ | \ 4 \ \#3 \ | \ 5 \ \#5 \ |
\end{align*}
\]
In all Cadences whatsoever, where the Bass rises a 4th, or falls a 5th, Observe, that the 4th falls half a Note into a sharp Third, and the 5th a whole Note into a flat 7th.

There is another Cadence call'd the 7th and 6th Cadence, which counted but a half Clofe, and if the 6th is flat, is never used for a final Clofe, because it does not satisfy the Ear, like as when the Bass falls 5th, or rises a 4th, 'tis often introduced in a piece of Musick, as the Air may require; and when it ends any one part of a Piece, 'tis in orderto begin a new Movement or Subject: The 7th and sharp 6th may be used for a final Clofe, if the Design of the Composers requires it, but 'tis very rarely done.

The following Example will shew how both the 7th and b6th, and 7th and #6 are used.

\[
\begin{array}{c|c|c|c|c|c|c|c}
7 & 8 & 8 & b7 & \#3 & 4 & 4 & #3 \\
5 & 6 & 5 & 5 & 8 & 8 & 8 & b7 \\
\end{array}
\]
Rules for a Thorow-Bass.

Observe when a Discord happens in the higher Note, leave the Cord out above it:

- The Flat 6 2 4 here instead of the 6th, the
- 4th and 2 4 6 2 5th may be added, but
- 4th and 2 4 6 then it ought to be marked second.

The Flat 6 b3 b5 here the 8th is o-
- 5th and 6 3 6 3 joined unless it be
- 6th join'd. 3 b5 6 in passing Notes. 5 join'd.

The perfect fifth when joined with a sixth is used like a Discord.

Example.

The Sharp 4 7 2 4, here to play When the 3d 7 3 4 4th, the 7th Sharp 6th
- 7th, when 4 4 7 2 full the Flat 6th may be added, but
- 4th and 4 7 6th may be marked above when it ought to
- lie flat.

The 9th resolving into 1 8 3 5 9 8, the 6th and 4th 6 2 4 6 6th and 4th 6 8 1 4
- 6th and 5 4 7 5 when the Bass 2 4 6 skips or lies flat.

Example.

The perfect fifth when joined with a sixth is used like a Discord.

Example.
The 7th and 5th happening just before the Cadence Note, here instead of the third, the ninth if prepar’d may be used but then it ought to be mark’d.

The extreme Flat seventh \{ b7 | 3 | b5 \} the extreme Flat seventh is and Flat fifth happening just \{ b5 | b7 | 3 \} the same with the Sharp before the Cadence Note. \{ b5 | b7 | 6 \} sixth.

The extreme Flat second \{ 6 | #2 | 4 \} the extreme Sharp second \{ 4 | #2 | 6 \} the extreme Sharp second is the same distance as the Flat third. \{ Ex. as follows. \}

\begin{align*}
\text{The 4th and 9th} & \{ 4 | 3 | 9 | 8 | 5 \} \text{ The 9th and 7th} & \{ 9 | 8 | 7 | 6 | 3 \} \\
\text{resolving into} & \{ 9 | 8 | 5 | 4 | 3 \} \text{ resolving into} & \{ 7 | 6 | 3 | 9 | 8 \} \\
\text{the 3d and 8th.} & \{ 5 | 4 | 3 | 9 | 8 \} \text{ the 3d and 8th.} & \{ 3 | 9 | 8 | 7 | 6 \}
\end{align*}

Example.

When the Bafs ascends or descends one or two Notes, move your Hands in playing contrary one to the other. After a sixth where no eighth is concerned, you may either ascend or descend together.
Rules for a Thorow-Bass.

After a sixth where the eighth lies in the middle, you may either ascend or descend.

Example.

Example of passing Notes in Common-time.
Rules for a Thorow-Bafs.
Rules for a Thorow-Bals.

Example of passing Notes in Triple-time:
Of Natural Sixes.

Play common Chords on all Notes where the following Rules don't direct you otherwise.

The natural Sixes in a Sharp Key are on the half Note below the Key, the third above the Key, and on all extraordinary Sharp Notes out of the Key, if not to the contrary mark'd, or prevented by Cadences.

The natural Sixes in a Flat Key are on the Note below the Key; the Note above the Key, and on all extraordinary Sharp Notes out of the Key, if not to the contrary mark'd, or prevented by Cadences.

When the Bass either in a Flat, or a Sharp Key, ascends or descends half a Note, Sixes are proper on the first Note, falling on the second, unless prevented by a Cadence.

When the Bass either in a Flat or Sharp Key, descends with a common Chord by thirds; Sixes are proper on the falling thirds.

When the Bass either in a Flat or a Sharp Key, ascends with a common Chord by thirds: Sixes are proper on the rising thirds. In a Flat Key the third above the Key generally requires a sixth to prepare the Cadence, the fifth being repugnant to the half Note below the Key.

Seldom two Notes ascend or descend but one of them hath a Sixth.
Example of Natural Sixes and proper Cadences in a sharp Key.
Example of Natural Sixes and proper Cadences in a flat Key.

\[ \text{Music notation image} \]
Now all these Síxes mentioned either in a Flat or Sharp Key, are not only to be observed in the Key you play in, but likewise in all other Cadences you are going into: And for the time you keep in that Cadence, observe the Rules for Síxes as tho' you were in the Key your Lesson is Composed in.

Where the Bafí ascends a perfect fourth, or descends a perfect fifth, Síxes are generally left.

Other Rules for Síxes are where the Bafí moves by degrees downwards, then these Síxes may be play'd on every other Note.

**Example.**

```
\[\text{Example.}\]
```

The Composer (especially in few parts) may Compose as many Síxes either ascending or descending by degrees as they think fit, but then they ought to be marked.

**Example.**

```
\[\text{Example.}\]
```

Now
Now here follows an Example where two Sixes are absolute necessary, and that descending because they are short Cadences instead of 7. 6 mark'd.

In a flat Key Descending.

In a sharp Key Descending.

Ascending.

Ascending.

When
Rules for a Thorow-Bafs.

When the Bafs lies still, the Seventh is generally resolved into the 6th, and the 9th into the 8th. The Example which follows, shews how Discords may be Resolved several ways.

Example.

Several Examples of what may be done when the Bafs Descends by degrees.

The Common way.

Natural and Artificial.
Rules for a Thorow-Bafs.

All Artificial.

When the Bafs Ascends by Degrees.

Example of all sorts of Discords in a flat Key.
Rules for a Thorow-Bais.

Where the Figures are set in Parenthesis, those I would have only dropped to set off Playing.

Example of all sorts of Discords in a sharp Key.
Rules for a Thorow-Bafs.
Rules for a Thorow-Bass.
To make some Chords easy to your memory, you may observe as follows: A common Chord to any Note makes a 3d, 6th, and 8th. to the 3d above it, or 6th. below it. A common Chord makes a 4th, 6th, and 8th. to the 5th. above it, or a 4th. below it. A common Chord makes a 3d, 5th. and 7th. to the 6th. above it, or a 3d. below it. A common Chord makes a 4th, 6th. and 2d. to the 7th. above it, or a 2d. below it.

A 2d. and 4th marked makes a common Chord to the Note above it, observing the 5th. perfect or imperfect, according to the Key, as also an 8th, 3d. and 6th. to the 4th. above it, or 5th. below it. A Sharp 7th. marked, where the Bafs lies flat makes 8th, 3d. and 6th. to the Note above it, and 5th, 7th. and Sharp 3d. to the Note below it. An extream Sharp 2d. and 4th. marked on a Flat Note, makes Sharp 3d. 5th. and 7th. on the half Note below it, as also a Sharp 6th. 8th. and 3d. to the Sharp 4th above it, or Flat 5th below it; the Flat 5th. and extream Flat 7th. marked on a Sharp Note, makes 3d. Flat 5th. and 8th. to the 3d. above it, or the 6th. below it, as also an 8th, 3d. and 6th. to the Flat 5th. above it, or Sharp 4th. below it. The 4th or 9th. mark'd is the perfect 5th. 6th. and 3d. on the whole Note below it, and the Flat 5th. 6th. and 3d. on the half Note below it, as also 3d. 7th. and 9th. to the 3d. above it, or 6th. below it, the 9th. and 7th. mark'd is the 5th, 9th. and 4th. on the 3d. below it, and the 6th. 3d. and perfect 5th. to the perfect 4th. below it, or the 5th. above it, and 6th. 3d. and Flat 5th. on the perfect 4th. below it.

The Flat 5th. and Sharp 4th. the extream Sharp 2d. and Flat 3d. The extream Flat 7th. and Sharp 6th. upon any fretted Instruments, or Harp-ficord. without quarter Notes, are the same thing in distance, yet the distinction is as follows.
There are some other Chords of the same kind, viz. the extrem flat 4th. being the same as the sharp 3d. and the extrem sharp 5th. the same as a flat 6th. but these Chords are only useful in three Parts, and will not admit a 4th. the distinction is as follows,

This extrem flat 4th. admits a 6th. for the second Part and is resolv'd into the third, and the 6th. into the flat 5th. The extrem sharp 5th. admits for the second Part a third.

Of Transposition.

Before any one can pretend to Transpose from one Key into another, it is necessary they shou'd know all the Flats and Sharps naturally belonging to all, at least the Practicable Keys.

Additional flats and sharps in Order.

The reason why I call the Flats and Sharps One, Two, Three, &c. is because where B is flat E may not, but where E is flat B must. The same reason holds good for sharps.
Next it is requisite to be acquainted with the several Cliffs and their removes, and last observe, that all Flat Keys must be Transpos'd into Flat Keys, and sharp into sharp ones: Which Keys are known according to their 3d. which is either Flat or Sharp.

**Rules for a Thorough-Bass.**

**C major Cliffs. C flat Cliffs. G solreut C. The Natural sharp Key.**

### A Note higher.

### A sharp 3d. higher.

### A flat 3d. higher.

### A 4th. higher.

### A 5th. higher.

### A sharp 6th. higher.

### A flat 6th. higher.

### A flat 7th higher.

### In a flat Key, the Natural.

### A 2d. lower.

### A flat 3d. lower.

### A sharp 3d. lower.
You are to observe what Flats or Sharp, belong to all the Keys, and imagine the Cliff that puts you in the Key you have a mind to Play in; and what you find too high or too low, according to the Compass of the Instrument you play on, you must Transpose an 8th higher, or lower which is easy enough to be done.

Of Discords, how many ways they may be prepar'd and resolv'd.

The 4th. when joyn'd with a 5th. or 6th. and is generally resolv'd into the third, may be prepar'd by a 3d. 5th. 6th. or 8th.
The 4th. prepar'd by a 6th. and Resolv'd into a 3d.

The 4th. prepar'd by an 8th. and Resolv'd into a 3d.

The 4th. on occasion Resolv'd into a 6th.

The 4th Resolv'd into the 3d. several times before you come to the Cadence.

The 7th. may be prepar'd by a 3d. 5th. 6th. 7th. or 8th. The 7th. when the Bass lies still Resolves into the 6th. and when the Bass falls Five Notes, or rises four Notes it Resolves into the 3d. The 7th. some times Resolves into a 5th. and then it is in order to a Cadence; so that the Bass rises one Note. I have seen the 7th. Resolv'd into an 8th. but it sounds so like two 8ths, that it makes me utterly against it.

The 7th. prepar'd by a 3d. and Resolv'd into a 6th.

The 7th. prepar'd by a 5th. and Resolv'd into a 6th.
The 7th. prepar'd by a 6th. and Resolv'd into a 6th.

Example of the 7th. Resolving into the 3d. or the 5th. some times.

There is sometimes two 7ths. Compos'd one after another, but it is call'd a Licence in Musick and commonly in order to a Cadence.

The 9th. is generally prepar'd by a 3d. or a 5th. and it may be by a 6th. or 8th. but not so naturally. The 9th when the Bass lies still, Resolves into the 8th. The 9th. when the Bass falls a 3d. Resolves into a 3d. The 9th. when the Bass rises a 3d. Resolves into a 6th. The 9th. may Resolve into a 5th. but not so naturally as the other, and then the Bass rises four Notes.
Rules for a Thorow-Bals.

The 9th, prepar'd by a 6th, and Resolv'd into the 8th.

The 9th, prepar'd by the 8th, and Resolv'd into the 8th.

of the 9th. Resolving into the 3d and 6th but rarely into a 5th.

Example:

The Flat 4th and 2d. and Sharp 4th. and 2d. is brought in when the Bass in a driving Note descends a half or whole Note, the Sharp 4th always Resolves into the 6th, as does generally the Flat 4th, but some times with the Flat 5th. the 2d. Resolves into the 3d.

Where the several driving Notes descend by degrees:

Example:

Another Example:
The 9th. and 7th. mark'd above one another, may be prepar'd by the 3d. and 8th. and Resolv'd into the 8th. and 6th. the Bass lying still and some times is artificially into the Flat 5th. and 3d. and the Bass falls a Flat 5th.

Example.

The 4th. and 9th mark'd one above another is best prepar'd by the 3d. and 5th. and Resolv'd into the 3d. and 8th. the Bass lying still, some times artificially into the Flat 5th. and 3d. the Bass falling a 3d. and some times into a 7th. the Bass rising four Notes contrary to the Trea-

Example.

The 4th. and 3d. mark'd one above another is commonly us'd when the Bass ascends by degrees, the 6th. and 4th. with a 2d. is commonly us'd when the Bass descends by degrees.

Example.
Rules for a Thorow-Bafs.

The 6th. and 4th. where the 8th. is joyn'd is commonly us'd when the Bafs lies still in a Sharp Key, or when the Bafs either descends four Notes, or ascends five Notes. Example.

The Sharp 7th. when accompany'd with a 2d. and 4th. is us'd, when the Bafs lies still in a Flat Key. Example.

The extream Sharp 2d. and 4th. generally prepares a Cadence. The 5th. and 7th. and the Flat 5th. and extream Flat 7th. are generally the fore runners of a Cadence. Example.
The perfect 5th. and 6th. joyn'd is commonly us'd as the 7th. and 5th. before a Cadence, as also when the Bafs descends by 3ds.

The Flat 5th. may be joyn'd to any Sharp Note that requires a 6th. unless it be contrary to the Key, or it be mark'd otherwise.

The extreme Sharp, and the extreme Flat Notes belonging naturally to either Flat or Sharp Key: The extreme Sharp in a sharp Key, is the half Note below the Key: The extreme Sharp in a flat Key, is the Note above the Key, unless taken off by an additional Sharp: The extreme Flat in a Sharp Key, is a 4th. above or the 5th. below the Key: The extreme Flat in a flat Key, is a 3d. below or a 6th. above the Key.

The extreme Sharp being too harsh, and the extreme Flat too luscious unless taken off by an additional Sharp or Flat, or what is excepted in the following Rules ought to be doubled.

On either extreme Sharp or Flat Note, or any extraordinary Sharp or Flat Note out of the Key, that requires a common Chord, you double the 8th. in Composition, or Playing four Parts. If the extreme Sharp or an extraordinary sharp Note requires a natural Flat 6th. you leave out the 8th. in four parts, and Compone, or Play two Sixes and one third or two thirds according as the Bafs is too high, or too low.

If the extreme Flat or any extraordinary flat Note requires a 6th. instead of double Sixes, or double thirds, you may Compone, or Play in four Parts.

Where the extreme Flat, or an extraordinary flat Note happens to make a 6th. to any Note, never double that Sixth.

Example in a Sharp Key.

\[\text{Example in a Sharp Key.}\]
Rules for a Thorow-Bafs.

Example in a Flat Key.
Some Lessons where the F. and the C. Cliffs Interfere one with the other.
Rules for a Thorow-Bass.
Rules for a Thorow-Bals.

In this Lesson the G, C, and F. are all us'd.
Rules for a Thorow-Bass.
Rules for a Thorow-Bass.
Rules for a Thorow-Bass.
I shall here add some short Lessons by way of Fugeing, to make the whole work Compleat.
Rules for a Thorow-Basses.
Rules for a Thorow-Bafs.
Rules for a Thorow-Bafs.
Rules for Tuning a Harpsicord or Spinet.

Tune the C-fol-fa-ut by a Confort Pitch-pipe.

Observe all the Sharp Thirds must be as sharp as the Ear will permit; and all Fifths as flat as the Ear will permit.

Now and then by way of Trypt, touch Unison Third, Fifth, and Eights; and afterward Unison Fourth and Sixth.
This book must not be taken from the Library building.